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<td>Doria-Cerezo, Arnau; Universitat Politècnica de Catalunya, Dept. of Electrical Engineering and IOC Batlle, Carles; Universitat Politècnica de Catalunya, Dept. of Applied Mathematics IV and IOC Espinosa, Gerardo; Universidad Nacional Autónoma de México, Facultad de Ingeniería</td>
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Passivity-based control of a bidirectional wound-rotor synchronous motor

Arnau Dória-Cerezo, Carles Batlle, and Gerardo Espinosa-Pérez

Department of Electrical Engineering and Institute of Industrial and Control Engineering, Universitat Politècnica de Catalunya, EPSEVG, 08800 Vilanova i la Geltrú, Spain

Department of Applied Mathematics IV and Institute of Industrial and Control Engineering, Universitat Politècnica de Catalunya, EPSEVG, 08800 Vilanova i la Geltrú, Spain

Facultad de Ingeniería, Universidad Nacional Autónoma de México, 04510 México DF, México

This paper presents a new nonlinear passivity-based controller for a wound rotor synchronous machine, acting as a motor drive. The control objectives are stated in the \(dq\)-frame, and the port-controlled Hamiltonian model is also obtained. A power flow analysis allows to state the control goals in terms of reactive power compensation and ohmic losses reduction. From the Hamiltonian structure, the Simultaneous Interconnection and Damping Assignment (SIDA-PBC) technique is used to compute the control action, which results in a controller with a simpler architecture than the standard one for this class of machines, able to cope with both positive and negative external mechanical loads and having thus bidirectional power capabilities. The robustness of the control action is also taken into account in the design procedure. The computed globally asymptotically stabilizing controller is validated via numerical simulations.

Keywords: passivity-based control; Simultaneous IDA-PBC; electrical machines; wound rotor synchronous motor

1 Introduction

The wound rotor synchronous machine (WRSM) is used for generation and also for drive applications (Rossi et al. (2006)). In the generation case the field voltage is used to regulate the stator voltage, while for motor applications this variable can be used to compensate the power factor of the machine (Schaefer (1999)). Several techniques for the control of the WRSM have been proposed in the literature. Linear techniques are the most used in the industry (see Leonhard (1995) or Senesky et al. (2004)), but decoupling methods (Ho and Sen (1995)), widely employed for asynchronous machines, have also been extended to the synchronous case, and advanced nonlinear controllers have been applied to this class of machines as well (Magri et al. (2006)). For many applications a bidirectional power flow is required. An example can be found in Hybrid Electrical Vehicles, HEV, (Zeraoulia et al. (2006)), were the regenerative breaking (employed to store kinetic energy into the batteries) is achieved using the electrical machine as a generator, by consigning a negative torque.

The main advantage of using a WRSM in front of the permanent magnet synchronous motor (PMSM), is the ability to compensate the reactive power consumed by the electrical machine. This feature can be done by selecting the appropriate rotor flux using the field voltage.

Usually, synchronous motors are controlled using two control loops. An inner current loop is designed to regulate the \(dq\) components of the stator current, and one or more outer loops provide the desired values of current to achieve the control goals (mechanical speed and reactive power). The proof of stability of this scheme is usually performed by invoking a time scale...
separation between the dynamics of the electrical and mechanical variables. In this paper, using passivity-based techniques, the speed controller is moved into the inner loop, the stability of which is proved rigorously in a passivity-based framework.

Passivity-based control (PBC) is a technique that can be used to design controllers for a large kind of systems. Control of a rather general class of electrical machines using PBC methods has been proposed in Nicklasson et al. (1997), and the specific cases for synchronous generators and drives can be found also in Espinosa-Pérez et al. (1997) and Guo et al. (2007), respectively. Recently, a new technique based on the PBC properties called Interconnection and Damping Assignment (IDA-PBC) has been proposed in Ortega et al. (2002). Using the IDA-PBC approach many electrical machines have been controlled (Rodriguez and Ortega (2003)), in particular induction machines (see example in Batlle et al. (2005)) and permanent magnet synchronous ones (Petrovic et al. (2001)). The simultaneous IDA-PBC methodology was proposed in Batlle et al. (2009), were the induction machine was studied and controlled. The SIDA-PBC technique offers more degrees of freedom than IDA-PBC, and allows to solve more complex interconnected systems and to design output feedback, as opposed to state feedback, controllers (see examples in Batlle et al. (2009) and Dória-Cerezo (2006)).

The main goal of this work is to design a control algorithm for a wound rotor synchronous drive machine, based on the Simultaneous IDA-PBC technique. The paper is organized as follows. In Section 3 the wound rotor synchronous machine model is introduced and its control goals are described. Section 4 presents the SIDA-PBC technique and then the control law is obtained. The simulation results are included in Section 5 and, finally, conclusions are stated in Section 6.

Throughout the paper * designs fixed point values while \( d \) is used for desired regulation values.

2 SIDA-PBC technique

The Simultaneous Interconnection and Damping Assignment (SIDA-PBC) method was proposed in Batlle et al. (2009) as a generalization of the Interconnection and Damping Assignment (IDA-PBC) technique (Ortega et al. (2002), Ortega et al. (2001)). It considers the problem of designing an state space controller for the stabilization of a desired equilibrium point of a nonlinear system

\[ \dot{x} = f(x) + g(x)u. \]

The key idea behind the SIDA-PBC technique (as it occurs also in the classic IDA-PBC) is to match the closed-loop dynamics to a PCHS form

\[ f(x) + g(x)u = F_d(x)\partial H_d, \]

where the left hand side may be given in explicit PCHS (van der Schaft and Maschke (1992), Dalsmo and van der Schaft (1998)) form \((J(x) - R(x))\partial H(x) + g(x)u\).

To enforce dissipativity, the following constraint on the \( F_d \) matrix is required

\[ F_d(x)^T + F_d(x) \leq 0. \]

The equilibrium assignment of the desired energy function translates to

\[ x^d = \arg \min H_d(x). \]

In (1) one has \( x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \), with \( m \leq n \) and, in practice, \( m < n \). Furthermore, the \( m \) columns of the \( n \times m \) matrix \( g(x) \) are assumed to be independent (if they are not one just has an overparametrization of the control input \( u \)), which is equivalent to \( g^T g \) being invertible. If \( m < n \), there is a basis of \( n - m \) left vectors of \( g \), which can be arranged row-wise in a matrix \( g^\perp \) such that \( g^\perp g = 0 \). Operating with \( g^\perp \) on the left of (1) one gets the so-called matching equation

\[ g^\perp(x)f(x) = g^\perp(x)F_d(x)\partial H_d(x), \]
which is either a partial differential equation for $H_d$ (Ortega et al. (2002)) with the additional requirement (3) if $F_d$ is given, or and algebraic equation for $F_d$ (Fujimoto and Sugie (2001)) subjected to (2), if a suitable $H_d$ is provided, or a mix of both (see Ortega and Garcia-Canseco (2004) for a survey). In this paper the algebraic approach will be adopted. With appropriate $F_d$ and $H_d$ choices, the control is obtained from (1) as

$$u = (g(x)^T g(x))^{-1} g^T(x) (F_d(x) \partial H_d(x) - f(x)),$$

where the invertibility of $g^T g$ has been employed. Note that the closed loop system defined by

$$\dot{x} = F_d(x) \partial H_d$$

can be easily written in a PCHS form

$$\dot{x} = (J_d(x) - R_d(x)) \partial H_d(x)$$

with

$$J_d(x) = \frac{F_d(x) - F_d(x)^T}{2}, R_d = \frac{F_d(x) + F_d(x)^T}{2}.$$  

3 The WRSM model and control goals

In this section we present the dynamical model of the WRSM. From the well-known dynamical equations we also propose a port-Hamiltonian model which allows to describe in a compact form and with a nice physically interpretation the system dynamics. Although the SIDA-PBC technique does not require to have the model in PCHS form, it can help in the design process. Finally, we compute, in terms of the fixed point values, the active and reactive powers flowing through the stator side of the machine and define the control objectives.

Figure 1 shows the scheme connection of a wound rotor synchronous machine acting as a motor.

3.1 dq-model of the WRSM

The $dq$ transformation (see Krause et al. (2002)) is commonly used to transform a tracking problem into a regulation one. The steady-state for the three phase electrical variables is given by periodic orbits which can be transformed into equilibrium points of a new two phase $dq$-system.

The state space model in $dq$ coordinates of a wound rotor synchronous machine with a field
winding (and no damper windings) is given by Chiasson (2005)

\[
\begin{align*}
\dot{\lambda}_d &= -R_s i_d + n_p \omega L_s i_q + v_d \\
\dot{\lambda}_q &= -n_p \omega L_s i_d - R_s i_q - n_p \omega M i_F + v_q \\
\dot{\lambda}_F &= -R_F i_F + v_F
\end{align*}
\]  \tag{9, 10, 11}

where \( \lambda^T = (\lambda_d, \lambda_q, \lambda_F) \in \mathbb{R}^3 \) and \( i^T = (i_d, i_q, i_F) \in \mathbb{R}^3 \) are the fluxes and currents, respectively\(^1\), \( \omega \) is the mechanical speed, \( n_p \) is the number of pole pairs, \( R_s \) and \( R_F \) are the ohmic resistances of the stator \( dq \) and rotor field windings, and \( L_s, L_F \) and \( M \) are the leakage and mutual inductances.

The dynamical model has to be completed with the mechanical equation

\[ J_m \frac{d\omega}{dt} = n_p M i_F i_q - B_r \omega + \tau_L. \]  \tag{12}

Here \( J_m \) is the rotor inertia, \( \tau_L \) is a generic mechanical torque (negative in case of braking), and \( B_r \) is a viscous mechanical damping coefficient.

Fluxes and currents are related by \( \lambda = L x \) where

\[
L = \begin{pmatrix}
L_s & 0 & M \\
0 & L_s & 0 \\
M & 0 & L_F 
\end{pmatrix}
\]  \tag{13}

3.2 Port-Hamiltonian model of the WRSM

Port-Hamiltonian modeling uses the state dependent energy functions to characterize the dynamics of the different subsystems, and connects them by means of network relations, such as Kirchhoff’s laws in the case of circuit theory or the two source-independent Maxwell equations for electromagnetism, which embody the power preserving physical laws. The result is a mathematical model with an specific structure, called port-controlled Hamiltonian system (PCHS, van der Schaft (2000)), which lends itself to a natural, physics-based analysis and control design (Kugi (2001), Ortega et al. (2001)).

Explicit PCHS have the form

\[
\begin{align*}
\dot{x} &= (J(x) - R(x)) \partial_x H(x) + g(x) u \\
y &= g^T(x) \partial_x H(x)
\end{align*}
\]  \tag{14}

where \( x \in \mathbb{R}^n \) is the vector state, \( u, y \in \mathbb{R}^m \) are the port variables, and \( H(x) : \mathbb{R}^n \rightarrow \mathbb{R} \) is the Hamiltonian function, representing the energy function of the system. The \( \partial_x \) (or \( \partial \), if no confusion arises) operator defines the gradient of a function of \( x \), and in what follows we will take it as a column vector. \( J(x) \in \mathbb{R}^{n \times n} \) is the interconnection matrix, which is skew-symmetric \( (J(x) = -J(x)^T) \), representing the internal energy flow in the system, and \( R(x) \in \mathbb{R}^{n \times n} \) is the dissipation matrix, symmetric and, in physical systems, semi-positive definite \( (R(x) = R^T \geq 0) \), which accounts for the internal losses of the system. Finally, \( g(x) \in \mathbb{R}^{n \times m} \) is a port matrix describing the port connection of the system to the outside the world. It yields the flow of energy to/from the system through the port variables, \( u \) and \( y \).

Equations (9), (10), (11) and (12) can be given a port-Hamiltonian form using as energy variables the fluxes \( \lambda_d, \lambda_q, \lambda_F \) and the momentum \( (p = J_m \omega) \). Then, the interconnection and

\(^1\) \( d, q \) and \( F \) subindexes refers to \( dq \) coordinates and the field variables, respectively.
dissipation matrices are, respectively

\[
J(x) = \begin{pmatrix}
0 & n_p L_s \omega & 0 & 0 \\
-n_p L_s \omega & 0 & 0 & -n_p M i_F \\
0 & 0 & 0 & 0 \\
0 & n_p M i_F & 0 & 0
\end{pmatrix},
\]

(15)

\[
R = \begin{pmatrix}
R_s & 0 & 0 & 0 \\
0 & R_s & 0 & 0 \\
0 & 0 & R_F & 0 \\
0 & 0 & 0 & B_r
\end{pmatrix},
\]

(16)

with Hamiltonian (energy) function

\[
H(x) = \frac{1}{2} \lambda^T L^{-1} \lambda + \frac{1}{2} J_m p^2,
\]

(17)

and with port matrix

\[
g = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix},
\]

(18)

where the external inputs available for control are \( u = (v_d, v_q, v_F) \). The external mechanical torque \( \tau_L \) will be treated as a perturbation of the model, and hence the PCHS with dissipation and added perturbation to which the SIDA-PBC technique will be applied is

\[
\dot{x} = (J - R) \partial H + gu + g_L \tau_L,
\]

(19)

with \( g_L = (0 \ 0 \ 0 \ 1)^T \). The left null vector of \( g \) is given by \( g^\perp = (0 \ 0 \ 0 \ 1)^T \).

### 3.3 Fixed points and power flow study

In this subsection we compute, in steady-state, the reactive power flowing through the stator side of the machine (in order to compensate the power factor) and we also establish the power balance of the machine. Throughout this study we consider that the three-phase system is sinusoidal and balanced.

Fixed points are solutions to

\[
0 = -R_s i_d^* + n_p \omega^* L_s i_q^* + v_d
\]

(20)

\[
0 = -n_p \omega^* L_s i_q^* - R_s i_d^* - n_p \omega^* M i_F^* + v_q
\]

(21)

\[
0 = -R_F i_F^* + v_F
\]

(22)

\[
0 = n_p M i_F^* i_q^* - B_r \omega^* + \tau_L.
\]

(23)

In this set of equations, \( \omega^* \) will be fixed to the desired value \( \omega^d \), while the desired values of \( i_F^* \) and \( i_d^* \) will be computed from additional power and quality requirements. With \( \tau_L \) considered as an external perturbation, the four equations above can be used to compute \( i_q^* \) and the steady
state values of $v_d$, $v_q$ and $v_F$. In dq-coordinates, the reactive power reads

$$Q_s = v_d i_q - v_q i_d,$$  \hfill (24)

and using the fixed points (20) and (21), we obtain

$$Q_s^* = -n_p \omega^*(L_s (i_{d*}^2 + i_{q*}^2) + M i_{d*}^* i_{q*}^*).$$  \hfill (25)

Notice that a suitable value of $i_F^*$ can compensate the power factor, i.e. $Q_s^* = 0$.

A power balance study is next performed to determine the optimal $i_d^*$ current in order to minimize the electrical losses $P_l$. Those can be computed as the difference between the electrical power supplied to the machine ($P_s$ and $P_F$) and the delivered mechanical power,

$$P_l^* = P_s^* + P_r^* - P_m^*.$$  \hfill (26)

From the definition of the stator dq-active power,

$$P_s = v_d i_d + v_q i_q,$$  \hfill (27)

and using the fixed points (20) and (21), we obtain

$$P_s^* = R_s (i_{d*}^2 + i_{q*}^2) + n_p \omega^* M i_{d*}^* i_{q*}^*.$$  \hfill (28)

The rotor power can be easily computed using

$$P_F = v_F i_F$$  \hfill (29)

which, in steady state and using (22), is rewritten as

$$P_F^* = R_F i_{F*}^2.$$  \hfill (30)

Finally, the generated mechanical power is defined as the product of the electrical torque and the mechanical speed,

$$P_m^* = \tau_e^* \omega^*.$$  \hfill (31)

Putting together (28), (30) and (31) in (26) and taking into account $\tau_e = n_p M i_F i_q$, we recover the electrical losses,

$$P_l = R_s (i_{d*}^2 + i_{q*}^2) + R_F i_{F*}^2.$$  \hfill (32)

Let us introduce the polar coordinates, $I$ and $\delta$, such that

$$i_{d*} = I^* \cos \delta^*$$  \hfill (33)

$$i_{q*} = I^* \sin \delta^*.$$  \hfill (34)

Taking into account (23) and considering that the reactive power goal $Q_s^* = 0$ is achieved, which in polar coordinates is

$$n_p L_s I^* \omega^* + \frac{(B r \omega^* - \tau_e)}{\sin \delta^*} \cos \delta^* = 0,$$  \hfill (35)
equation (32) can be rewritten as

$$P_l = -\frac{(B_r \omega^* - \tau_L)}{n_p} \left( \frac{R_s \cos \delta^*}{L_s \sin \delta^*} \right) + \frac{R_F L_s}{M^2} \frac{1}{\sin \delta^* \cos \delta^*}$$

(36)

This function has a minimum at

$$\delta^{op} = \pi - \arccos \left( -\sqrt{\frac{R_F L_s^2}{2 R_F^2 L_s^2 + R_s M^2}} \right),$$

which implies, with (35) and (33), that

$$i_d^{op} = I^{op} \cos \delta^{op}$$

(37)

where

$$I^{op} = \sqrt{\frac{(B_r \omega^* - \tau_L) \cos \delta^{op}}{n_p L_s \sin \delta^{op}}}.$$  

(38)

Note that $\delta^{op}$ only depends on the electrical parameters, while the mechanical ones $B_r$ and $\tau_L$ appear in $I^{op}$. In order to obtain an expression which does not depend on the mechanical parameters, (38) can be rewritten, with the help of (23), as

$$I^{op} = \sqrt{\frac{M}{L_s} i_d^{op} \frac{\cos \delta^{op}}{i^{op} \sin \delta^{op}}}.$$  

(39)

### 3.4 Control objectives

Following the results presented above, we can summarize the control goals as follows: to regulate the mechanical speed at a desired value $\omega^d$, and to compensate the power factor, i.e. $Q^d_s = 0$, and also to minimize loses regulating $i_d$ at the optimal point $i_d^{op}$. To achieve these objectives we have three control inputs, namely $v_d$, $v_q$ and $v_F$.

### 4 Control design

Figure 2 shows the proposed control scheme. As explained above the control objectives are to regulate $\omega^* = \omega^d$, to compensate the reactive power and to regulate $i_d$ at the optimal value in order to minimize losses ($i_d^{op} = i_d^{op}$). The complete controller consists has an inner-loop which regulates the $i_d$ and $i_F$ current and the mechanical speed, and an outer-loop which computes the required value of $i_F$ to compensate the reactive power.

A passivity-based controller, using the Simultaneous IDA-PBC approach, is designed for the inner-loop. The obtained control law ensures asymptotic stability. This new design also improves the one presented in Batlle et al. (2008), since there is no restriction on the maximum values of the control gains, and also the control law allows for negative $i_F$ values.

The reactive power is regulated with the Q Controller block in Figure 2. The design of this outer-loop is based on the assumption that the inner-loop is much faster than the factor power compensation dynamics. Figure 3 shows the structure of the standard architecture for this machine (Chiasson (2005)). The Speed Controller of the classical architecture has been moved to the SIDA-PBC Controller in our scheme, and instead of the Back-EMF Compensator, the controller is designed in order to minimize the ohmic losses by means of the selection of $i_d^{op}$, and this has been incorporated also into the SIDA-PBC Controller block.
In this Section, we first present the inner-loop control design, using SIDA-PBC technique, and finally we propose a control law for the power factor compensation.

4.1 SIDA-PBC for the WRSM

As proposed in Section 2, we will use the SIDA-PBC approach to design the inner-control loop for the WRSM. First we fix the energy function as

\[ H_d = \frac{1}{2} (x - x^*)^T P (x - x^*) \]  

(40)

where \( P = P^T > 0 \), \( x^T = (i_d, i_q, i_F, \omega) \) and \( x^* = (i_{d_{op}}, i_{q_{op}}, i_{F_{op}}, \omega^d) \). To simplify the solution we restrict \( P \) to have the form

\[
P = \begin{pmatrix}
k_d & 0 & 0 & 0 \\
0 & \gamma_q & 0 & 0 \\
0 & 0 & \gamma_F & 0 \\
0 & 0 & 0 & \gamma_\omega
\end{pmatrix}.
\]

(41)

Notice that the positiveness requirement on \( P \) implies that \( k_d, \gamma_q, \gamma_F, \gamma_\omega > 0 \).

The \( F(x) \) matrix is chosen in order to facilitate the solution of the resulting algebraic equations. Furthermore, in order to simplify the control structure, we want to assign one output to each control action and also to obtain a controller as simpler and robust (minimum parameter dependence) as possible. The proposed structure is as follows

\[
F(x) = \begin{pmatrix}
F_{11}(x) & 0 & 0 & 0 \\
0 & 0 & 0 & F_{24}(x) \\
0 & 0 & F_{33}(x) & 0 \\
0 & F_{42}(x) & F_{43}(x) & F_{44}(x)
\end{pmatrix},
\]

(42)

which must also satisfy the inequality (2).

Decoupling control actions and feedback outputs is possible if the \( F \) matrix contains only one nonzero element in each row. The three first rows of \( F(x) \) have nonzero \( F_{11}, F_{24} \) and \( F_{33} \) elements,
which relate \( v_d, v_q \) and \( v_F \) to the errors in \( i_d, \omega \) and \( i_F \), respectively. This is a critical point to improve the robustness of the resulting controller, because in this way only the fixed points of the outputs appear in the control action. Indeed, as it is shown later, the fixed point \( i_q^* \), whose computation requires solving the fixed points equations, (20)–(23), with strong dependence on the parameters, does not appear in the control law.

Acting with \( g^T \) on (19), which selects the three first rows, one obtains the control actions

\[
\begin{align*}
v_d &= R_s i_d - n_p \omega L_s i_q + F_{11} k_d (i_d - i_d^*) \\
v_q &= n_p \omega L_s i_d + R_s i_q + n_p \omega M i_F + F_{24} \gamma (\omega - \omega_d) \\
v_F &= R_F i_F + F_{33} \gamma_F (i_F - i_F^*) ,
\end{align*}
\]

while operating with \( g^\perp \), which in this case singles out the fourth row, one gets the matching equation

\[
n_p M i_F i_q - B_r \omega + \tau_L - F_{42} \gamma (i_q - i_q^*) - F_{43} \gamma_F (i_F - i_F^*) - F_{44} \gamma (\omega - \omega_d) = 0.
\] (46)

Equation (46) can be solved with

\[
\begin{align*}
F_{42} &= \frac{1}{\gamma_q} n_p M i_q^d \\
F_{43} &= \frac{1}{\gamma_F} n_p M i_q \\
F_{44} &= -\frac{1}{\gamma_{\omega}} B_r
\end{align*}
\]

where the steady-state solution of (12)

\[
\tau_L = -n_p M i_q^d i_q^* + B_r \omega_d
\] (50)

has been also used. This means that we assume that the dynamics of \( i_q \) is much faster than the characteristic time of variation of the external load torque; notice however that we are not supposing anything with respect to the characteristic time of the mechanical variable \( \omega \).

In order to simplify the solution, now we assign

\[
\begin{align*}
F_{11} &= -1 \\
F_{24} &= -F_{42}.
\end{align*}
\]

With this choice, it is clear that (2) holds if \( F_{33} < 0 \) and

\[
4 F_{33} F_{44} - F_{43}^2 > 0,
\] (53)

or substituting,

\[
-4 F_{33} \frac{B_r}{\gamma_{\omega}} - \left( \frac{1}{\gamma_F} n_p M i_q \right)^2 > 0.
\] (54)

Finally, with

\[
F_{33} = -\frac{1}{4} n_p^2 M^2 i_q^* \leq 0
\] (55)
the previous equation reduces to

\[
\gamma \omega < B_r \gamma_F^2. \tag{56}
\]

To further simplify the controller the following parameters are assigned

\[
\gamma_q = \frac{16 k_F^2}{n_p M^3 k_\omega \epsilon}, \tag{57}
\]

\[
\gamma_F = \frac{4}{n_p M^2 k_F}, \tag{58}
\]

\[
\gamma_\omega = \frac{\gamma_q n_p M k_\omega \epsilon}{n_p M k_\omega}, \tag{59}
\]

where \(\epsilon > 0\) is a remaining free design parameter. With this the controller becomes

\[
v_d = R_s i_d - n_p \omega L_s i_q - k_d (i_d - i_d^*), \tag{60}
\]

\[
v_q = n_p \omega L_s i_d + R_s i_q + n_p \omega M i_F - i_F^2 k_\omega (\omega - \omega^*), \tag{61}
\]

\[
v_F = R_F i_F - k_F i_F^2 (i_F - i_F^d), \tag{62}
\]

while the stability condition (56) simplifies to \(\epsilon < B_r\). However, \(\epsilon\) does not appear in the control gain ranges, and so it is not even necessary to know a lower bound for \(B_r\) in order to implement the above controller. The condition on \(\epsilon\) is hence only necessary to prove stability, and it relies on \(B_r > 0\), which is a sensible assumption for any real machine. With this, stability of the inner-loop is ensured for any \(k_d, k_F, \omega > 0\) values. Notice that the stability of this inner loop is what in the literature is discussed in terms of the time-scale separation of electrical and mechanical dynamics, and that one of the main contributions of the present paper is to solve this question using a single, more rigorous step.

The controller (60)(61)(62) contains a part that looks like a classic feedback linearization, but the field control action \(v_F\) introduces a nonlinear term \(k_F i_F^2 (i_F - i_F^d)\).

### 4.2 Reactive power control loop

As mentioned above, the power factor is compensated via the field current \(i_F\), see equation (25). The proposed control law to achieve the power factor compensations consists in

\[
i_F^d = \frac{k_i}{i_F^* \omega_d} \int (Q_s - Q_s^d) dt. \tag{63}
\]

The stability of the complete closed-loop system can be argued taking into account that the inner loop is faster than the \(Q_s\) outer loop. Then, considering that the \(i_d, i_q, i_F\) and \(\omega\) reaches its desired values \((i_d^o, i_q^o, i_F^o, \omega^o)\), from (24) the \(Q_s\) dynamics yields

\[
\dot{Q}_s = -n_p M \omega^d i_d \frac{d i_F^d}{dt}. \tag{64}
\]

Finally, the closed loop dynamics is obtained replacing (63),

\[
\dot{Q}_s = -k_i n_p M (Q_s - Q_s^d), \tag{65}
\]

which clearly stabilizes at \(Q_s = Q_s^d\).
5 Simulations

In this section we present some simulations using the designed controller in Section 4. The WRSM parameters are: $L_s = 1\text{mH}$, $R_s = 0.0303\Omega$, $M = 1.5\text{mH}$, $L_F = 8.3\text{mH}$, $R_F = 0.0539\Omega$, $n_p = 2$, $J_m = 0.01525\text{kg\cdot m}^2$, $B_r = 0.05\text{N\cdot m\cdot s}$ and $\tau_L = 1\text{N\cdot m}$. The control parameters are selected as: $k_d = 50$, $k_F = 1$, $k_\omega = 0.05$ and $k_i = 200$, and the desired reactive power is, in all cases, set to zero, $Q^d_s = 0$.

The first numerical experiment is performed increasing the desired speed from $\omega^d = 200\text{rad\cdot s}^{-1}$ to $\omega^d = 250\text{rad\cdot s}^{-1}$ at $t = 0.01s$. For these values, the optimal value of $i_d$, computed in (37) changes from $i_d^{op} = 5.32\text{A}$ to $i_d^{op} = 5.88\text{A}$. Figure 4 shows that the system is perfectly regulated under changes of the desired outputs.

![Figure 4. Simulation results: Mechanical speed, reactive power and $i_d$ current, under a change of the speed reference.](image)

Figure 5 shows the results of a second test. In this case the external torque is suddenly changed at $t = 0.01$ from $\tau_L = -1\text{N\cdot m}$ to $\tau_L = 1.5\text{N\cdot m}$. Now the optimal value of $i_d$ goes from $i_d^{op} = 5.32\text{A}$ to $i_d^{op} = 3.5\text{A}$. Note that this torque change also involves a change of the power sign in the stator side, and the WRSM is now generating electric energy. Figure 6 shows the behaviour of the stator active power and the bidirectional ability of the proposed controller.

6 Conclusions

The SIDA-PBC technique has been applied to control a wound rotor synchronous machine for a motor drive application. The SIDA-PBC matching equation has been solved using the algebraic approach and the desired robustness of the resulting controller has been taken into account. The obtained controller has a simpler architecture than the standard one, is globally asymptotically stable, assures stability for a large range of control gain values, and works for either positive or
negative load torque, thus displaying bidirectional power behavior. The stability of the inner-loop has been proved rigorously using passive, port-Hamiltonian based techniques, while an additional outer loop that takes into account the power factor regulation has been introduced and the stability of the complete controller has been discussed in terms of separation of time scales. The presented method also allows to decouple the outputs, improving the robustness and facilitating the gain tuning.

Future research includes a dynamical extension keeping the Hamiltonian structure to improve the robustness (basically on the electrical parameters $R_s$, $L_s$, $M$, and $R_F$) and the performance. This can be easily done for the $i_d$ and $i_F$ currents (due to the fact that they are passive outputs), but a more complicated task is to design a dynamical extension for the speed part. Experimental validation with a real plant using the control law designed in this paper will be also considered in the future.
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