Experimental Evaluation of a Cascade Sliding Mode-PI Controller for a Coupled-Inductor Boost Converter

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Abstract—This paper describes the experimental results obtained for a cascade sliding mode PI control in a coupled-inductor Boost converter. A non-linear control strategy similar to sliding mode control was designed for the inner loop, while for the secondary control loop an experimentally tuned PI controller is proposed. The main goal of the closed loop system is to regulate the output voltage of the converter. The control was designed starting with a piece-wise complementarity model of the converter. The complementarity framework allows to model uni-directional switches, with the idealized current and voltage of the diodes as “complementary variables”, i.e. variables that are non-negative and such that their product is zero at any time. The experimental results show the robustness of the control strategy under step changes in the input voltage and load.

I. INTRODUCTION

There are several power converters architectures that yield high conversion rates with minimal losses and that are suitable for applications requiring high power. [1] proposed a step-up converter with coupled inductors, able to provide high conversion rate without extreme duty cycles and with high efficiency, since the problems of reverse recovery time of the diode are mitigated. Other variants of this converter appear in [2]. Although the total number of components of the converter is low and its topology is relatively simple, obtaining generalized mathematical models is not a trivial task due to the passive switches (diodes), which introduce new modes in one switching period, in addition to the modes caused by the active switch. As a consequence, issues such as the stability analysis and design of control strategies for these converters are also quite complex.

Linear complementarity systems (LCS) constitute an appropriate framework to obtain the dynamic models of power converters that contain several ideal diodes and switches [3], [4], [5]. The main advantage of using LCS is that it captures all modes of the converter, without having a priori knowledge of the succession of modes or topologies during one switching cycle and of the instants that the topology changes occur. Furthermore, a single model is able to represent the converter operation in both continuous conduction mode (CCM) and discontinuous conduction mode (DCM) operations. A piece-wise complementarity model of the coupled-inductor boost converter was proposed in [6]. With a single set of differential equations this model is able to describe the hybrid dynamics in both transient and steady state operating in DCM.

Up to our knowledge, there are not general results in the literature dealing with control theory for complementarity systems. In [7], a general framework for the construction of complementarity models of closed loop power converters is presented, and several specific cases of voltage-controlled and current-controlled DC-DC converters are considered. On the other hand, in [8] a cascade sliding mode PID control for a coupled-inductor boost converter was proposed. The control strategy was developed using a piece-wise complementarity model of the converter. The simulation results of the closed-loop demonstrated the effectiveness of the approach. This paper focuses on the experimental validation of the correctness of this modelling and control approach. The robustness properties of the designed control law will become evident upon the introduction of step changes in the input voltage and load.

The paper is organized as follows. Section II gives a brief account of the dynamic model of the coupled-inductor boost converter using the complementarity framework [6], while the control law proposed in [8] is presented in Section III. A real scenario about the prototype of the converter and the control law is described in Section IV. Section V focuses on the experimental results. Finally, some conclusions and considerations about further research are presented in Section VI.

II. COMPLEMENTARITY DYNAMIC MODEL

The circuit configuration of the proposed coupled-inductor boost converter is shown in Fig. 1. According to [6], [8], the normalized model of such converter using LCS framework can be written by the following set of differential equations

\[
\begin{align*}
\dot{x} &= A_1 x + B_1 \omega + E_1 V_{in} + (A x + B \omega + E V_{in}) u \\
\dot{z} &= C_1 x + D_1 \omega + F_1 V_{in} + (C x + D \omega + F V_{in}) u \\
0 &\leq \omega \perp z \geq 0
\end{align*}
\]

\[u = \begin{cases} 
1 & \rightarrow \ S = \text{On} \\
0 & \rightarrow \ S = \text{Off}
\end{cases}
\]
where $A = A_2 - A_1$, $B = B_2 - B_1$, $C = C_2 - C_1$, $D = D_2 - D_1$, $E = E_2 - E_1$, $F = F_2 - F_1$ and $u \in \{0, 1\}$ represents the switch state. For each switch state the matrices and the complementary variables are given by

- $S=\text{Off} \ (i_2 = 0)$

$$A_1 = \begin{pmatrix} 0 & 0 & -L_2 & M \\ 0 & 0 & M & -L_1 \\ 1 & 0 & -\Delta & -\Delta \\ 0 & 1 & -\Delta & -\Delta \end{pmatrix}, \quad B_1 = \begin{pmatrix} -L_2 & M \\ M & -L_1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$E_1 = \begin{pmatrix} L_2 & -M \\ -M & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$D_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad F_1 = \begin{pmatrix} 0 & 0 \end{pmatrix}.$$  

and

$$\omega_1 = -v_{D1}, \quad z_1 = i_{D1} = x_1$$

$$\omega_2 = -v_{D2}, \quad z_2 = i_{D2} = x_2$$  

- $S=\text{On} \ (i_2 = 0)$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & M \\ 0 & 0 & 0 & -L_1 \\ 0 & 0 & -\Delta & -\Delta \\ 0 & 1 & -\Delta & -\Delta \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & M \\ 0 & -L_1 \\ -1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$E_2 = E_1, \quad C_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$D_2 = D_1, \quad F_2 = F_1$$  

and

$$\omega_1 = -v_{D1}, \quad z_1 = -v_{D1} = x_3$$

$$\omega_2 = -v_{D2}, \quad z_2 = i_{D2} = x_2$$

The normalized state variables are related to the physical ones by $x_1 = \frac{1}{L_1} i_{L1}$, $x_2 = \frac{1}{L_2} i_{L2}$, $x_3 = \frac{1}{C_1} v_{C1}$, and $x_4 = \frac{1}{C_2} v_{C2}$ denote the inductor currents through $L_1$ and $L_2$ and the voltages across the capacitor $C_1$ and $C_2$ respectively, and $k = L_1 L_2 - M^2$, $\tau = \frac{k}{2A\Delta}$, $\Delta = \frac{V_r}{k}$, $\Gamma = \sqrt{\frac{k}{\Delta}}$. The diodes $D_1$ and $D_2$ are modelled as ideal diodes, and their current-voltage characteristics are expressed by the pair of complementary variables $z$ and $\omega$, which obey the “complementary conditions” (CC) $0 \leq \omega \perp z \geq 0$ [4]. These inequalities hold component-wise, and $\perp$ denotes the orthogonality between both vectors, so that $z$ can be positive only if $\omega = 0$, and vice versa. Once the state of the switches is fixed by the control algorithm, the complementarity formalism takes care of the topology changes induced by the diodes, without a priori knowledge of them.

### III. Control design

For DC-DC converters the main control goal is to regulate the output dc-voltage to a given reference value. In [8] a cascade control architecture, whose basic structure is shown in Fig. 2, was proposed. The primary control loop is similar to a sliding mode controller with the goal of controlling the fast dynamics of the converter (the inductor currents). For the secondary control loop a PI control is introduced in order to regulate the slow dynamics (output voltage).

#### A. Current control design

The sliding surface

$$\sigma(x) = L_1 x_1 + M x_2 - \sigma^*$$  

where $x_1$ and $x_2$ are the inductor currents and $\sigma^*$ represents the reference value defined by the secondary control loop PI control, was proposed in [8]. The discontinuous control law that locally forces the state trajectories to reach the sliding surface is given by

$$u = \begin{cases} 0 & \text{if } \sigma(x) > \delta \\ 1 & \text{if } \sigma(x) < -\delta \end{cases}$$

where $\delta$ defines an hysteresis band around the sliding surface. The reader is referred to [8] for further details about how this current controller is designed from the complementary variables model of the system.

#### B. Voltage control design

A PI controller is used for the secondary loop in order to regulate the output dc-voltage to a reference value. The PI control block in Figure 2 is thus given by

$$\sigma^*(t) = K_P e(t) + K_I \int_0^t e(t) dt$$  

where $e(t) = V_{ref} - V_R$ and the parameters $(K_P, K_I)$ were designed through experimental tuning according to different performance criteria such as minimal overshoot and shorter settling time. The values of the parameters of the PI controller are given in the next section.

### IV. Experimental setup

Fig. 3 shows the experimental setup used to validate the proposed control, while Fig. 4 depicts a picture of the actual circuit. The system is composed of an electronic prototype of the coupled-inductor boost converter, sensors signal conditioning module, DSP, the circuit control and switch driver. The nominal values of the converter are: $V_{in} = 12V$, $L_1 = 75\mu H$, $C_1 = 100\mu F$, $C_2 = 220\mu F$, and $V_{ref} = 100V$. The parameters of the PI controller were tuned experimentally to $K_P = 100$, $K_I = 10$.
$L_2 = 525 \mu H, M = 196 \mu H, C_1 = 22 \mu F, C_2 = 22 \mu F, R = 113 \Omega$.
The diodes $D_1$ and $D_2$ are ultrafast soft recovery 60EPU02 60A 200 Vrrm diodes, and the switch is implemented using a MOSFET IRFPS40N50L N-CH 500V 46A. In addition this circuit has a 2.2nF capacitor connected in parallel to the switch in order to minimize voltage ringing between the drain and source terminals of the MOSFET, and an identical capacitor is always in the range supported by the ADC.

Taking the reference voltage and the proportional and integral actions of the PI controller goal is to regulate the output dc-voltage to a value $V_{ref} = 120V$ starting from initial conditions equal to zero. The transient and steady state of the output voltage (blue trace) and the current through the inductor $L_1$ (magenta trace) trajectories are shown in Fig 6. The results show that the output voltage is able to reach the desired value after a brief transient in a stable way. Fig. 7 shows the ac-component of the output voltage.

The control law was also tested for several switching frequency values. In particular, Fig. 9 shows again the steady state trajectories of the inductor currents $L_1$ (magenta trace) and $L_2$ (green trace), sliding surface (red trace) and the activation signal or control action $u$ (blue trace). From the surface trajectory it can be observed that the existence and reaching conditions to have sliding mode on $s(x) = 0$ are satisfied. The switching frequency is about 40KHz and the duty cycle is $d = 74\%$.

In order to test the robust tracking of the control law, two different step disturbances were introduced. The first one corresponds to a change in the nominal value of the load from $R = 113 \Omega$ to $R = 213 \Omega$ and vice versa. The waveforms of the output voltage $V_o$ (blue trace) and inductor current $L_1$ (magenta trace) are given by $K_p = R_{ps} / R_{u}$ and $K_i = 1 / (R_{C_1})$. The switching is implemented using the AD746 and OP4676 by Analog Devices. The sliding function used in the electronic implementation is given by

$$\sigma(x) = R_{s1} \left( \frac{v_{L1}}{R_{s1}} + \frac{v_{L2}}{R_{s2}} - \frac{v^*}{R_{s3}} + \frac{\bar{v}}{R_{s4}} \right)$$

where $v^*$ is the reference value given by the PI controller. In addition, the sliding surface has an offset value $\bar{v} = 2.5V$ in order to make this signal less susceptible to noise. A non-inverting Schmitt trigger is used to implement a small hysteresis band around the sliding surface. This control action is implemented using a high speed operational amplifier LM311 whose output square signal is in the range $0 - 5V$. This latter signal turns the switch of the converter on/off.

### V. EXPERIMENTAL EVALUATION

The main purpose of this section is to discuss the experimental validation of the designed control law. The controller is used to regulate the output dc-voltage to a value $V_{ref} = 120V$ starting from initial conditions equal to zero. The transient and steady state of the output voltage (blue trace) and the current through the inductor $L_1$ (magenta trace) trajectories are shown in Fig 6. The results show that the output voltage is able to reach the desired value after a brief transient in a stable way. Fig. 7 shows the ac-component of the output voltage.

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The control law was also tested for several switching frequency values. In particular, Fig. 9 shows again the steady state trajectories of the inductor currents $L_1$ (magenta trace) and $L_2$ (green trace), sliding surface (red trace) and the control action $u$ (blue trace) for $f = 74$KHz. As it can be seen from the surface trajectory, increasing the switching frequency can lead to multiple switches during the period of time in which the switch is on, due to the new slope that appears at the beginning of the switching period. However, this effect is not observed in the simulation results presented in [8]. These differences may be due to the quality of the signals measured and observed, to parasitic elements which have not been considered or to unmodeled dynamics that might get excited.

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where $v^*$ is the reference value given by the PI controller. In addition, the sliding surface has an offset value $\bar{v} = 2.5V$ in order to make this signal less susceptible to noise. A non-inverting Schmitt trigger is used to implement a small hysteresis band around the sliding surface. This control action is implemented using a high speed operational amplifier LM311 whose output square signal is in the range $0 - 5V$. This latter signal turns the switch of the converter on/off.
trace) trajectories respectively are shown in Fig. 10. As it can be seen, the control law is able to reject the load perturbation in less than 10ms despite of a small overshoot that appears when the change is applied. The second test corresponds to a step change in the nominal value of the input voltage from $V_{in} = 12V$ to $V_{in} = 10V$ and vice versa. Fig. 11 shows the waveforms of the output voltage $V_R$ (blue trace) and inductor current $L_1$ (magenta trace) trajectories for this test. It can be noticed that the output voltage trajectory has a little overshoot but it quickly attains steady state at the desired value.

VI. CONCLUSIONS

Experimental results of a cascade control law for a coupled-inductor boost converter have been presented. The experimental results demonstrate the effectiveness, excellent performance and stable tracking of the obtained controller, applied under the influence of perturbations at input voltage and load. The proposed control law has a cascade architecture with two loops, one based on sliding mode control for the inner loop and a PI controller for the outer loop. This design of the inner loop controlled was performed using a piece-wise complementarity model of the converter, which takes into account its hybrid dynamics.
Fig. 6. Output voltage $V_R$ (blue) and inductor current $L_1$ (magenta) trajectories. (Scales: CH3= 2V/div, CH4= 20V/div).

Fig. 7. Ac-component of the output voltage $V_R$. (Scale: CH4= 500mV/div).

Fig. 8. Experimental trajectories in steady state of the inductor currents, sliding surface and control action. (Scales: CH1= 5V/div, CH2= 1V/div, CH3= 1V/div, CH4= 2V/div).

Fig. 9. Experimental trajectories in steady state of the inductor currents, sliding surface and control action for $f = 74$KHz. (Scales: CH1= 5V/div, CH2= 2V/div, CH3= 2V/div, CH4= 2V/div).

(a) $R$ from 113Ω to 213Ω.

(b) $R$ from 213Ω to 113Ω.

Fig. 10. Load change test: Output voltage $V_R$ (blue) and inductor current $L_1$ (magenta) trajectories. (Scales: CH3= 1V/div, CH4= 20V/div).
Future research will include a general analysis of the ideal sliding dynamics for linear complementarity systems and stability analysis under sliding mode. The design of robust and efficient control for linear complementarity systems is still an interesting and open problem.

REFERENCES


