IOC 17013 Exercises — Lecture 2

1. Consider a feedback system with

\[ P(s) = \frac{1}{10s + 1}, \quad C(s) = k, \quad F(s) = 1. \]

Find the least positive gain \( k \) such that the following conditions hold simultaneously

- The feedback system is internally stable.
- \( |e(\infty)| \leq 0.1 \) when \( r(t) \) is the unit step and \( n = d = 0 \).
- \( \|y\|_{\infty} \leq 0.1 \) for all \( d(t) \) such that \( \|d\|_{2} \leq 1 \), with \( r = n = 0 \).

2. Consider the standard feedback system with \( F = 1, \ r = n = 0 \) and \( d(t) = \theta(t) \sin \omega t \), and assume that it is internally stable. Show that

\[ \lim_{t \to +\infty} y(t) = 0 \]

iff either \( P \) has a zero at \( s = j\omega \) or \( C \) has a pole at \( s = j\omega \). If possible, check this a Simulink simulation for a particular example. \textit{Hint:} Remember the condition under which the final value theorem holds.

3. Consider a feedback system with plant \( P \) strictly proper and sensor \( F \) proper. Find conditions on \( P \) and \( F \) for the existence (you do not have to compute it) of a proper controller so that, simultaneously,

- (a) the feedback system is internally stable,
- (b) \( y(t) - r(t) \to 0 \) when \( r \) is a unit step, and
- (c) \( y(t) \to 0 \) when \( d \) is a sinusoid of frequency 100 rad s\(^{-1}\).