On the Metric Dimension of Cartesian Products of Graphs

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Abstract

A set $S$ of vertices in a graph $G$ \textit{resolves} $G$ if every vertex is uniquely determined by its vector of distances to the vertices in $S$. The \textit{metric dimension} of $G$ is the minimum cardinality of a resolving set of $G$. This paper studies the metric dimension of cartesian products $G \Box H$. We prove that the metric dimension of $G \Box G$ is tied in a strong sense to the minimum order of a so-called doubly resolving set in $G$. Using bounds on the order of doubly resolving sets, we establish bounds on $G \Box H$ for many examples of $G$ and $H$. One of our main results is a family of graphs $G$ with bounded metric dimension for which the metric dimension of $G \Box G$ is unbounded.

\textbf{Key words:} graph, distance, resolving set, metric dimension, metric basis, cartesian product, Hamming graph, Mastermind, coin weighing

1 Introduction

This paper undertakes a general study of the metric dimension of cartesian products of graphs. All the graphs considered are finite, undirected, simple,