Institute of Physics Publishing
Journal of Physics: Conference Series **43** (2006) 518–521
doi:10.1088/1742-6596/43/1/128
7th European Conference on Applied Superconductivity

Computation limits of Current Distribution in thick Superconducting Bulks from Magnetic Field Measurements

J. Amorós[1], M. Carrera[2], X. Granados[3], S. Iliescu[4], E. Moreno[1], B. Bozzo[3] and X. Obradors[3]

[1]Dep. Matemàtica Aplicada I. Universitat Politècnica de Catalunya, Diagonal 647, Barcelona, Spain [2] Dep. Medi Ambient i Ciències del Sòl, Universitat de Lleida, Jaume II, 69. 25001 Lleida, Spain [3] Institut de Ciència de Materials de Barcelona, CSIC, Campus UAB, 08193 Bellaterra, Spain [4] Dep. Ciència de Materials i Eng. Met. Universitat Politècnica de Catalunya, Diagonal 647, Barcelona, Spain

granados@icmab.es

Abstract. We present the latest progress of our computation method for critical currents in bulk YBCO samples from magnetic field measurements with Hall probe, based on discretization of the magnetization M and resolution of the linearized inverse Biot-Savart problem by QR inversion. While computationally harder than other linear/FFT methods, it is more robust in front of measurement errors, and allows the computation of detailed current maps without assuming any symmetry in the OXY plane. In particular, we are able to detect irregularities such as multiple domains, or the perturbation induced by a welding. If the current in the bulk sample is not homogenous along the OZ axis, we obtain the average along the OZ axis of the current density. Moreover, the inhomogeneity of the current along the vertical axis can be detected by the user by measuring the magnetic field on the side of the bulk sample and comparing it with that induced by the computed OZ-averaged current. Finally, we discuss what must be done in the measurement of the magnetic field and mathematical processing of the data to avoid numerical pitfalls, and discuss the limits to the computation of 3-dimensional current maps.

1. Introduction

We discuss in this note the computation of critical current density (Jc) maps in bulk YBCO samples with planar crystallization from measurements of the magnetic field B around the sample. Such a computation is required for quality control of macroscopic samples where single or multiple domains may exist, or weldings, where the quality of the weld is determined by the ratio between the in-grain and inter-grain currents, computed from the measurements of the magnetic field above the sample [1,2]. Jc maps computed at the web [3] are shown in both references.

After the earliest, simplest computation schemes that assumed complete symmetry in the geometry and current distribution in the sample, and estimated its density from a single central measurement of B, the use of computers and a discretization procedure first proposed by Xing et al. [4] has led to two practical procedures for the computation of 2-dimensional J_c maps on thin film samples of any size: the inverse Fourier transform method of Wijngaarden et al. [5,6], and the linearization and QR inversion method used by the authors, described in [7,8], and implemented in the

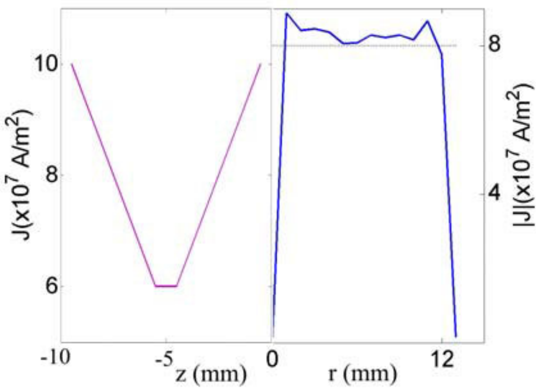

Figure 1. Radial distribution of the averaged current module along the z-axis (right) obtained by application of Caragol Inverse Problem solver from the magnetic field generated by the current density shown on the left. The dashed line shows the z-axis average $\mathbf{J_m}$ as deduced from that current distribution (Example 1).

software package Caragol, freely usable on the web [3]. Neither method assumes any symmetry or regularity on the geometry of the sample and its current distribution, the only constraint assumed in either computation is that the current must be homogeneus along the c-axis, and both yield detailed maps of J_c. The advantages of each method are those of its numerical algorithm: the inverse FFT method is fast (it works in real time for 200x200 element maps on real samples), the QR inversion method of Caragol is slower but far more robust in front of noise in measured data, yielding detailed distribution maps in samples where inaccurate measurement of **B** prevents a Fourier inversion.

Thus the frontier in critical current determination is now the computation of 3-dimensional **J** maps on bulk samples. The authors have applied both the Fourier and QR inversion procedures to bulk samples with planar crystallization by assuming that the current density **J** is homogeneous along the c-axis, that is, the current map is the same on all horizontal layers. Section 2 shows the results of these computations: the current $\mathbf{J_m}$(x,y) obtained by assuming homogeneity along the c-axis is the average along this axis of the true current **J**(x,y,z), with a slight deviation if the measurement of **B** is made only on one side of the sample. The fact that the computed current is an average and not the true value may be detected by measuring **B** close on the side of the sample.

It is reported in [9] that linear inversion schemes such as the two discussed above fail to compute 3-dimensional maps of **J**, or even the average $\mathbf{J_m}$(x,y). We show in section 3 that the failure of the inversion attempts in [9] stems from a problem that the authors had already warned about in [7]: measurement of the vertical magnetic field B_z(x,y) far above the sample makes the condition number of the resulting linear system [7] huge, ruining the inversion of the system. Nevertheless, we present more robust evidence against the possibility of computing 3-dimensional current maps from measurements of B_z(x,y) above the sample: example 2 in section 2 is a simulated sample with current varying in 2 layers, for which we find a linear inversion scheme with condition number good enough for inversion. Yet the computation fails to yield the correct values of current.

2. Critical current computation in thick samples

If we apply our QR inversion method to thick samples with a current distribution that is not homogeneous in the c-axis, the consequence of the c-homogeneity hypothesis made by our algorithm is that on every point (x,y) of the sample we obtain a value for the critical current $\mathbf{J_m}$(x,y) which is the average of the actual values **J**(x,y,z) with z ranging along the thickness of the sample. This is shown in our *Example 1*, in which we simulate a bulk sample with cylindrical shape (12.5 mm radius, 10 mm thickness) and a critical current **J** forming a regular domain on the cylinder but with density varying with z as shown in figure 1 (left).

We compute the vertical magnetic field B_z(x,y) that this current generates in a grid at a height of 0.3 mm above the sample, with relative error below $2\text{x}10^{-6}$, and apply the Caragol software to this magnetic field. The result is displayed in figure 1 (right): Caragol finds a regular domain whose c-homogeneous critical current $\mathbf{J_m}$(x,y) is the c-axis averaged value of the original current **J**(x,y,z), with a small increase in the 2-4% range because the upper sample layers (closer to the B_z-measurement grid) have current densities that are higher than the sample average. A similar result may be achieved by using Fourier instead of QR inversion.

The authors have checked that it is impossible to verify the homogeneity of the current density along the c-axis by studying the magnetic field in a layer above the sample. Even if one takes into account all components of the magnetic field **B**, the c-averaged current induces a magnetic field in the above layer that is indistinguishable from that of the original, non-homogeneous current **J**. Nevertheless, the c-axis inhomogeneity of the critical current on a given sample may be detected by measuring the magnetic field **B** in the sides of the sample: if the current is not homogeneous along the c-axis ($\mathbf{J} \neq \mathbf{J_m}$) a significant discrepancy of field values arises in the immediate neighbourhood of the sample. This is shown for our Example 1 sample in figure 2, which shows the difference between the magnetic fields induced by the non-c-homogeneous current density **J** and the c-axis average $\mathbf{J_m}$ around the sample: the areas where this difference is measureable are those coloured in black, they are a 1 mm strip around the sample (1 mm is the thickness of every layer with a sizeable change in the density J in Example 1), plus 1 mm thick regions at the endpoints of the cylinder axis. The regions at the ends of the axis are not useful in real samples because, as the error in the measured **B** grows, the small additional error in the computation of the c-average $\mathbf{J_m}$ that arises due to the singularity of **J** at the origin may mask the c-inhomogeneity error. However, the magnetic field **B** measured on the sides of the sample, as close as possible to it, is a reliable indicator of the c-homogeneity (or lack of) of the current density. This fact points the way to the computation of 3-d current maps, at least in samples with all domains reaching the boundary.

In [9] the author claims that it is impossible to find 3-dimensional current maps from measurements of the magnetic field above the sample, adducing as proof the high condition number of his resulting linear problem (in the range of 10^8). We will show in the next section that the high condition number in [9] is simply a consequence of the excessive height at which the magnetic field B_z is used. We provide now another example (henceforth called *Example 2*) of a simulated bulk sample with two layers and an inversion scheme that has condition number low enough to allow the numerical inversion of the system, yet any inversion scheme leads to a flawed solution.

The sample geometry is again a cylinder with the same dimensions as in Example 1, and a single, regular, current domain. The cylinder is divided in two horizontal layers with different current density: the top layer has thickness 1 mm and $|J|=6\times10^7$ A/m^2, while the bottom layer has thickness 9 mm and $|J|=10^8$ A/m^2. The magnetic field **B** is computed in rectangular grids at heights 0.1 and 1 mm above the sample, and the standard discretization and QR inversion procedure of Caragol is used for the inverse computation of the current density in both layers. The condition number of the resulting system is N=236.7, yielding a global relative error in the computed **M** of 5×10^{-4}. Yet the computation fails for the reason explained above and displayed in figure 2: a computation of **J** based on measurements of **B** taken above the sample cannot adjudicate correctly the distribution of current among the different layers of the sample. The computation adjudicates almost all current circulation to the bottom layer, resulting in a correct but overvalued current map, while the thin top layer is adjudicated almost no current, resulting in a completely erroneous value for the current density on this layer.

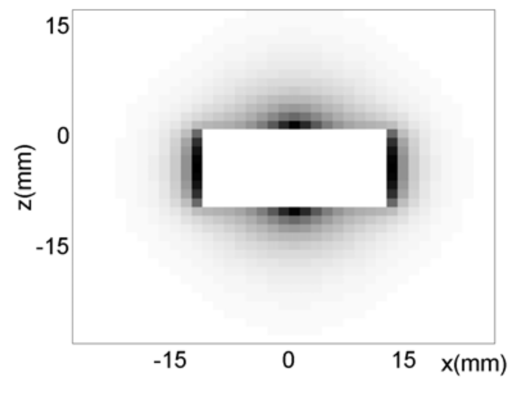

Figure 2. Difference between the magnetic fields induced by the current distribution **J** of figure 1(left) and its z-axis average $\mathbf{J_m}$ around the sample in a vertical cross-section.

3. Numerical discussion of the computation: how to measure the magnetic field data

The numerical analysis of current density computations such as the examples of section 2 has been made in [7,8], albeit omitting the error analysis of the replacement of the current **J**(x,y,z) by its c-average $\mathbf{J_m}$.

As shown in [7], the main source of error in our inverse Biot-Savart problems is the inaccuracy in the measurement of the magnetic field **B**, arising from background noise and the limited accuracy of the measuring probe. This error propagates to **M**, **J** multiplied by a factor called the condition number *N* of the system, according to the formula

$$||\Delta M||/||M|| < N\ ||\Delta B||/||B|| \qquad (1)$$

The right hand side product on formula (1) must be smaller than the admissible error in **M**. The computation of **J** from **M** typically doubles the relative error. We recall from [7,8] how to minimize the error coming from this source:

- Minimize background noise when measuring the magnetic field **B**.
- Measure the magnetic field **B** as close to the sample as possible, because the weaker the magnetic field becomes, the larger the relative error $||\Delta \mathbf{B}||/||\mathbf{B}||$ is, and the condition number *N* grows (over)exponentially with distance from measures to the sample.
- Make your linear system overdetermined, i.e. make the number of discretized values of **M** smaller than that of measures of **B**, until the condition number *N* of the resulting system becomes low enough for formula (1) to give an admissible error bound for the solution **M**.

We specially point out the step 2 to achieve a low condition number and thus a computation with less error. A theoretically sound way to compute 3-dimensional current maps on a thick sample is to measure the magnetic field **B** on a band above the sample with height commensurate to the thickness of the original sample [9]. However, such an approach guarantees failure because not only the magnetic field induced by the original current density distribution and that of its c-average distribution become indistinguishable above the sample except at a very small height (as illustrated by figure 2), but also the condition number of the system grows with the height of measure of B as illustrated by the table 1.

height (mm)	**0.1**	**0.3**	**1**	**2**	**5**	**10**
N	**1.86**	**3.99**	**70.8**	**4.5×10^3**	**1.2×10^9**	**1.1×10^{16}**

Table 1: Condition number vs height of measurement of the magnetic field in the sample of Example 1. Note that for a measurement of **B** within a 1% precision the error in the computed magnetization reaches 70% already at the height of 1 mm.

4. Conclusions

The computation of 3-dimensional maps of the current density distribution in a thick YBCO sample by linearization and QR or Fourier inversion of the Biot-Savart equation is not yet known to be an impossible task. The available methods, such as the software Caragol (usable at the site [3]) compute the c-axis average of the current density when applied to bulk samples. To achieve an accurate current map in 2 or 3 dimensions it is essential that the magnetic field **B** be measured as close to the sample as possible.

If the computation of 3-dimensional maps of current density by inverting the Biot-Savart problem is feasible, it will require measuring the magnetic field B not only above the sample but also in its sides and in any gap through it that its geometry allows.

Acknowledgment

This work was supported by the Spanish CICYT (MAT2002-02642, MAT2003-01584 and BFM2003-06001), and the Generalitat de Catalunya (Pla de Recerca 00206 and CeRMAE).

References

[1] X. Granados, B. Bozzo, S. Iliescu, E. Bartolomé, T. Puig, X. Obradors, J. Amorós and M. Carrera, IEEE Trans. Appl. Supercond. 15 (2) (2005) 3632.
[2] B. Bozzo, S. Iliescu, E. Bartolomé, A. Palau, X. Granados, T. Puig, X. Obradors, J. Amorós and M. Carrera, Supercond. Sci. Technol. 18 (2005) 1.
[3] Caragol, computer program for critical current computation, freely usable at http://jaumetor.upc.es/caragol
[4]W.Xing, B. Heinrich, Zhou Hu, A. Fife, A. Cragg. J. Appl. Phys. 76 (1994) 4244.
[5] R. J. Wijngaarden, H. J. W. Spoelder, R. Surdeanu, and R. Griessen, Phys. Rev. B 54 (1996) 6742.
[6] R.J. Wijngaarden, K. Heeck, H.J.W. Spoelder, R. Surdeanu, R Griessen, Physica C 295 (1998) 177.
[7] M. Carrera, J. Amorós, X. Obradors, J. Fontcuberta, Supercond. Sci. Technol. 16 (2003) 1187.
[8] M. Carrera, J. Amorós, A.E. Carrillo, X. Obradors, J. Fontcuberta, Physica C 385 (2003) 539.
[9] M. Eisterer, Supercond. Sci. Technol. 18 (2005) S58.