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Simple inflationary quintessential model. II. Power law potentials

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The present work is a sequel of our previous work [Phys. Rev. D **93**, 084018 (2016)] which depicted a simple version of an inflationary quintessential model whose inflationary stage was described by a Higgs-type potential and the quintessential phase was responsible due to an exponential potential. Additionally, the model predicted a nonsingular universe in past which was geodesically past incomplete. Further, it was also found that the model is in agreement with the Planck 2013 data when running is allowed. But, this model provides a theoretical value of the running which is far smaller than the central value of the best fit in n_s, r, $\alpha_s \equiv dn_s/dlnk$ parameter space where n_s, r, α_s respectively denote the spectral index, tensor-to-scalar ratio and the running of the spectral index associated with any inflationary model, and consequently to analyze the viability of the model one has to focus in the two-dimensional marginalized confidence level in the allowed domain of the plane (n_s, r) without taking into account the running. Unfortunately, such analysis shows that this model does not pass this test. However, in this sequel we propose a family of models runs by a single parameter $\alpha \in [0, 1]$ which proposes another "inflationary quintessential model" where the inflation and the quintessence regimes are respectively described by a power law potential and a cosmological constant. The model is also nonsingular although geodesically past incomplete as in the cited model. Moreover, the present one is found to be more simple compared to the previous model and it is in excellent agreement with the observational data. In fact, we note that, unlike the previous model, a large number of the models of this family with $\alpha \in [0, \frac{1}{2})$ match with both Planck 2013 and Planck 2015 data without allowing the running. Thus, the properties in the current family of models compared to its past companion justify its need for a better cosmological model with the successive improvement of the observational data.

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I. INTRODUCTION

In [1] the authors of the current work presented a cosmological background in a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe whose dynamics was characterized by the Raychauduri equation $\dot{H} = F(H)$ where $F(H)$ (H is the Hubble rate of the FLRW universe) was a linear function in H before the phase transition and after the phase transition it became quadratic in H. In summary, the model realized threefold properties: (i) The linear part prevented the big bang singularity in finite cosmic time although it was geodesically past incomplete, (ii) the phase transition was essential to produce enough particles to reheat the universe and the universe went through a deflationary period for a sufficient time, and finally (iii) the quadratic part had a fixed point that became responsible for the current acceleration of the universe. What was interesting of that background is that it comes from a quintessential potential, whose inflationary part was a Higgs-style potential and the quintessential part reads an exponential potential. However, the model has some undesired features, such as it provides a reheating temperature in the MeV regime, although it does not contradict the nucleosynthesis success since this needs a very low temperature, and the worse thing in the model is that, since the theoretical value of the running is far from the corresponding observational mean value obtained by Planck's team, thus, comparing the theoretical results provided by this model with Planck 2013 and Planck 2015 observational data [2,3] when the running is not allowed, one can show that the model has to be disregarded.

However, we found that the existing disparities in [1] can be defeated in a family of models in this flat FLRW background which has the same feature as in [1] but in an improved manner. Hence, in this sequel we propose a family of backgrounds whose dynamics before the phase transition is governed by the Raychaudhuri equation $\dot{H} = -k^2 H^\alpha$ with $\alpha \in [0, 1]$ and k is any real number. This family provides an inflationary quintessential potential whose inflationary part is basically a power law potential and the quintessential potential is governed by a cosmological constant. Further, the models of this family are

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nonsingular in nature. In other words, although the models are geodesically past incomplete, they do not encounter any finite time past singularity, i.e. big bang. Also, the models provide a complete analytic background similar to [1]. Further, due to the presence of the power law potential, the models provide a greater reheating temperature in the GeV or TeV regime depending on the value of the parameter α. Moreover, for some values of this parameter the models match correctly with Planck 2013 [2] and Planck 2015 data [3] without allowing the running, which indeed is an interesting and notable point in the present family of models.

The manuscript is organized as follows: In Sec. II, we introduce the family of backgrounds and discuss its properties. In Sec. III, we establish that the dynamics governed by the model could be mimicked by a single scalar field whose potential is a combination of a *power law potential*, and a *cosmological constant*. Section IV is devoted to the study of cosmological perturbations showing that the theoretical results provided by our models fit well with current observed data [2,3]. The reheating process via gravitational particle production of heavy massive particles is studied in Sec. V, where we show that our family of models provides a reheating temperature in the GeV and TeV regime depending on the value of the parameter α. A detailed calculation of the number of e-folds is performed in Sec. VI. In Sec. VII we compare our new family of models with the model proposed in our previous work [1]. Finally in Sec. VIII we have summarized our results.

We note that the units used throughout the paper are $\hbar = c = 1$.

II. THE MODEL

We start with the following dynamical equation:

$$\dot{H} = \begin{cases} (-3H_E^2 + \Lambda)\left(\frac{H}{H_E}\right)^{\alpha} & \text{for } H \geq H_E \\ -3H^2 + \Lambda & \text{for } H \leq H_E, \end{cases} \tag{1}$$

where H_E is a specific value of the Hubble parameter, $\Lambda \ll H_E^2$ is a positive cosmological constant and $\alpha \in [0, 1]$ is the parameter which defines the family of models under consideration. Now, Eq. (1) can analytically be solved leading to the following backgrounds:

1. For $\alpha = 1$ the Hubble parameter is given by

$$H(t) = \begin{cases} H_E e^{\frac{(-3H_E^2+\Lambda)t}{H_E}} & t \leq 0 \\ \sqrt{\frac{\Lambda}{3}}\frac{3H_E+\sqrt{3\Lambda}\tanh(\sqrt{3\Lambda}t)}{3H_E\tanh(\sqrt{3\Lambda}t)+\sqrt{3\Lambda}} & t \geq 0, \end{cases} \tag{2}$$

and thus, the scale factor can be solved as

$$a(t) \cong \begin{cases} a_E e^{\frac{H_E^2}{-3H_E^2+\Lambda}\left[e^{\frac{(-3H_E^2+\Lambda)t}{H_E}}-1\right]} & t \leq 0 \\ a_E\left(\frac{3H_E}{\sqrt{3\Lambda}}\sinh(\sqrt{3\Lambda}t)+\cosh(\sqrt{3\Lambda}t)\right)^{\frac{1}{3}} & t \geq 0. \end{cases} \tag{3}$$

2. For $0 \leq \alpha < 1$, the Hubble parameter has the following expression:

$$H(t) = \begin{cases} H_E((\alpha-1)(-3H_E+\frac{\Lambda}{H_E})t+1)^{\frac{1}{1-\alpha}} & t \leq 0 \\ \sqrt{\frac{\Lambda}{3}}\frac{3H_E+\sqrt{3\Lambda}\tanh(\sqrt{3\Lambda}t)}{3H_E\tanh(\sqrt{3\Lambda}t)+\sqrt{3\Lambda}} & t \geq 0, \end{cases} \tag{4}$$

and the corresponding scale factor is

$$a(t) = \begin{cases} a_E e^{\frac{H_E}{(-3H_E+\frac{\Lambda}{H_E})(2-\alpha)}[((\alpha-1)(-3H_E+\frac{\Lambda}{H_E})t+1)^{\frac{2-\alpha}{1-\alpha}}-1]} & t \leq 0 \\ a_E\left(\frac{3H_E}{\sqrt{3\Lambda}}\sinh(\sqrt{3\Lambda}t)+\cosh(\sqrt{3\Lambda}t)\right)^{\frac{1}{3}} & t \geq 0. \end{cases} \tag{5}$$

In all cases ($0 \leq \alpha \leq 1$), the family depicts a nonsingular background in cosmic time satisfying $H(-\infty) = \infty$, and $H(\infty) = \sqrt{\frac{\Lambda}{3}}$. Moreover, for $\Lambda \cong 0$, one can have the following approximate forms of the Hubble parameter and the scale factor:

1. For $\alpha = 1$:

$$H(t) \cong \begin{cases} H_E e^{-3H_E t} & t \leq 0 \\ \frac{H_E}{3H_E t+1} & t \gtrsim 0, \end{cases} \tag{6}$$

and

$$a(t) \cong \begin{cases} a_E e^{-\frac{1}{3}[e^{-3H_E t}-1]} & t \leq 0 \\ a_E(3H_E t+1)^{\frac{1}{3}} & t \gtrsim 0. \end{cases} \tag{7}$$

2. For $0 \leq \alpha < 1$:

$$H(t) = \begin{cases} H_E(3(\alpha-1)H_E t+1)^{\frac{1}{1-\alpha}} & t \leq 0 \\ \frac{H_E}{3H_E t+1} & t \gtrsim 0, \end{cases} \tag{8}$$

and

$$a(t) = \begin{cases} a_E e^{-\frac{1}{3(2-\alpha)}[(3(\alpha-1)H_E t+1)^{\frac{2-\alpha}{1-\alpha}}-1]} & t \leq 0 \\ a_E(3H_E t+1)^{\frac{1}{3}} & t \gtrsim 0. \end{cases} \tag{9}$$

On the other hand, the effective equation of state (EoS) parameter, namely $w_{\rm eff}$, which is defined as $w_{\rm eff} = -1 - \frac{2\dot{H}}{3H^2}$, for our family of models is given by

$$w_{\rm eff} = \begin{cases} -1 + 2\left(1 - \frac{\Lambda}{3H_E^2}\right)\left(\frac{H}{H_E}\right)^{\alpha-2} & H \geq H_E \\ 1 - \frac{2\Lambda}{3H^2} & H \leq H_E, \end{cases} \tag{10}$$

which shows that for $H \gg H_E$ one has $w_{\rm eff}(H) \cong -1$ (early quasi–de Sitter period). When $H \cong H_E$, the EoS parameter satisfies $w_{\rm eff}(H) \cong 1$ (kination or deflationary period [4,5]), and finally, for $H \cong \sqrt{\frac{\Lambda}{3}}$ one also has $w_{\rm eff}(H) \cong -1$ (late quasi–de Sitter period). Moreover, when one considers the approximation $\Lambda = 0$, the equation of state becomes

$$P = \begin{cases} -\rho + 2\left(\frac{\rho}{\rho_E}\right)^{\frac{\alpha-2}{2}}\rho & \rho \geq \rho_E \\ \rho & \rho \leq \rho_E, \end{cases} \tag{11}$$

where ρ, P are respectively the energy density and the pressure of the cosmic fluid and ρ_E is the energy density of the universe at $H = H_E$. In particular,

1. For $\alpha = 0$, the equation of state becomes

$$P = \begin{cases} -\rho + 2\rho_E & \rho \geq \rho_E \\ \rho & \rho \leq \rho_E. \end{cases} \tag{12}$$

2. For $\alpha = 1$, the equation of state takes the form

$$P = \begin{cases} -\rho + 2\sqrt{\rho\rho_E} & \rho \geq \rho_E \\ \rho & \rho \leq \rho_E. \end{cases} \tag{13}$$

III. THE SCALAR FIELD

It is evident from Eqs. (12) and (13) that at early times, our family of backgrounds satisfies $P(\rho) \cong -\rho$, which means our universe was quasi–de Sitter in nature. Since the dynamics of the early accelerating phase is well realized via a scalar field prescription, hence it is very natural to ask whether an equivalence between the family and the scalar field dynamics exists or not. If such an equivalence exists then we need to confirm their viability with the observational data, which means essentially we aim to check whether the family of models could lead to a power spectrum of cosmological perturbations that fit well with the current observational data [2,3]. To do so, in the flat FLRW universe, if we consider a scalar field φ with potential $V(\varphi)$, then the energy density and the pressure of the scalar field represented by the notations ρ_φ, p_φ respectively, assume the following simplest forms:

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \qquad p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi). \tag{14}$$

Now, using Eq. (14) and the Raychaudhuri equation $\dot{H} = -\frac{\dot{\varphi}^2}{2M_{\rm pl}^2}$ [where $M_{\rm pl}^2 = (8\pi G)^{-1}$ is the reduced Planck's mass], we find

$$\varphi = M_{\rm pl}\int \sqrt{-2\dot{H}}dt = -M_{\rm pl}\int \sqrt{-\frac{2}{\dot{H}}}dH. \tag{15}$$

Now, in our case the scalar field is solved as

$$\varphi = \begin{cases} \varphi_E\left(\frac{H}{H_E}\right)^{\frac{2-\alpha}{2}} & H \geq H_E \\ -\sqrt{\frac{2}{3}}M_{\rm pl}\ln\left(\frac{\sqrt{H^2-\frac{\Lambda}{3}}+H}{\sqrt{H_E^2-\frac{\Lambda}{3}}+H_E}\right) + \varphi_E & H \lesssim H_E, \end{cases} \tag{16}$$

where $\varphi_E \equiv -\frac{2\sqrt{2}}{\sqrt{3}(2-\alpha)}\frac{H_E}{\sqrt{H_E^2-\frac{\Lambda}{3}}}M_{\rm pl} \cong -\frac{2\sqrt{2}}{\sqrt{3}(2-\alpha)}M_{\rm pl}$.

Conversely, one can express the Hubble rate in terms of the field as

$$H = \begin{cases} H_E\left(\frac{\varphi}{\varphi_E}\right)^{\frac{2}{2-\alpha}} & \varphi \leq \varphi_E \\ \frac{\left(\sqrt{H_E^2-\frac{\Lambda}{3}}+H_E\right)^2 e^{-\sqrt{\frac{3}{2}}(\varphi-\varphi_E)} + \frac{\Lambda}{3}e^{\sqrt{\frac{3}{2}}(\varphi-\varphi_E)}}{2\left(\sqrt{H_E^2-\frac{\Lambda}{3}}+H_E\right)} & \varphi \geq \varphi_E. \end{cases} \tag{17}$$

The potential is given by $V(H) = 3H^2M_{\rm pl}^2 + \dot{H}M_{\rm pl}^2 \Rightarrow V(\varphi) = 3H^2(\varphi)M_{\rm pl}^2 + \dot{H}(\varphi)M_{\rm pl}^2$. Then, for our family one has

$$V(H) = \begin{cases} 3H^\alpha\left(H^{2-\alpha} - \frac{H_E^2-\frac{\Lambda}{3}}{H_E^\alpha}\right)M_{\rm pl}^2 & H \geq H_E \\ \Lambda M_{\rm pl}^2 & H \leq H_E. \end{cases} \tag{18}$$

That is,

$$V(\varphi) = \begin{cases} 3\left(\frac{H_E M_{\rm pl}}{\varphi_E}\right)^2\left(\frac{\varphi}{\varphi_E}\right)^{\frac{2\alpha}{2-\alpha}}\left[\varphi^2 - \varphi_E^2\left(1 - \frac{\Lambda}{3H_E^2}\right)\right] & \varphi \leq \varphi_E \\ \Lambda M_{\rm pl}^2 & \varphi \geq \varphi_E. \end{cases} \tag{19}$$

Note that, for $\alpha = 0$, the potential is quadratic, for $\alpha = \frac{2}{3}$, it is cubic and for $\alpha = 1$, it is quartic.

IV. COSMOLOGICAL PERTURBATIONS

Now, to study the cosmological perturbations, one needs to introduce the slow roll parameters [6]

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = 2\epsilon - \frac{\dot{\epsilon}}{2H\epsilon}, \tag{20}$$

which allow us to calculate the associated inflationary parameters, such as the spectral index (n_s), its running (α_s), and the ratio of tensor to scalar perturbations (r) defined below:

$$n_s - 1 = -6\epsilon_* + 2\eta_*, \qquad \alpha_s = \frac{H\dot{n}_s}{H^2 + \dot{H}}, \qquad r = 16\epsilon_*, \tag{21}$$

where the star $(*)$ means that the quantities are evaluated when the pivot scale crosses the Hubble radius. Now, for our family of models, the above inflationary parameters assume the following values:

$$n_s - 1 = (\alpha - 4)\epsilon_*, \qquad \alpha_s = \frac{(\alpha-4)(2-\alpha)\epsilon_*^2}{1-\epsilon_*}, \qquad r = 16\epsilon_*, \tag{22}$$

where $\epsilon_* = 3(\frac{H_E}{H_*})^{2-\alpha}$. Now, let us remark the following:

Remark 4.1: For potentials of the form $V(\varphi) = \lambda\varphi^{\frac{4}{2-\alpha}}$, and using that

$$\epsilon \cong \frac{M_{\rm pl}^2}{2}\left(\frac{V_\varphi}{V}\right)^2, \qquad \eta \cong M_{\rm pl}^2 \frac{V_{\varphi\varphi}}{V}$$

one also obtains that $n_s - 1 \cong (\alpha - 4)\epsilon_*$, which means that our family of potentials, during the inflationary regime, are like power law potentials.

The number of e-folds is given by

$$N = \int_{t_*}^{t_{\rm end}} H dt = -\int_{H_{\rm end}}^{H_*} \frac{H}{\dot{H}} dH = \frac{1}{2-\alpha}\left(\frac{1}{\epsilon_*} - 1\right). \tag{23}$$

Then, in terms of the number of e-folds one has

$$n_s - 1 = \frac{\alpha - 4}{1 + (2-\alpha)N}, \qquad r = \frac{16}{1 + (2-\alpha)N},$$
$$\alpha_s = \frac{\alpha - 4}{N(1 + (2-\alpha)N)}. \tag{24}$$

From this last formula and due to the large value of the number of e-folds, one can see that the running is of the same order as $(n_s - 1)^2$. Then, it is clear that its theoretical value is far smaller than the central value of the best fit obtained by Planck's team (see for instance Table 5 of [2]), and as a consequence to check the viability of our models we have to consider the two-dimensional marginalized confidence level in the plane (n_s, r) without the presence of running provided by Planck's team. Moreover, it is important to realize that in quintessential inflation, the number of e-folds is greater than the e-folds for inflationary potentials with a deep well [7]. For this reason, here we have drawn the curves from $N = 65$ to $N = 75$. Taking into account these considerations, we have shown in Fig. 1 that the models allowed by Planck 2015 data at 2σ C.L. must satisfy $\alpha \in [0, \frac{1}{2})$.

Finally, to determinate the value of H_E, one has to take into account the theoretical [6] and the observational [8] value of the power spectrum

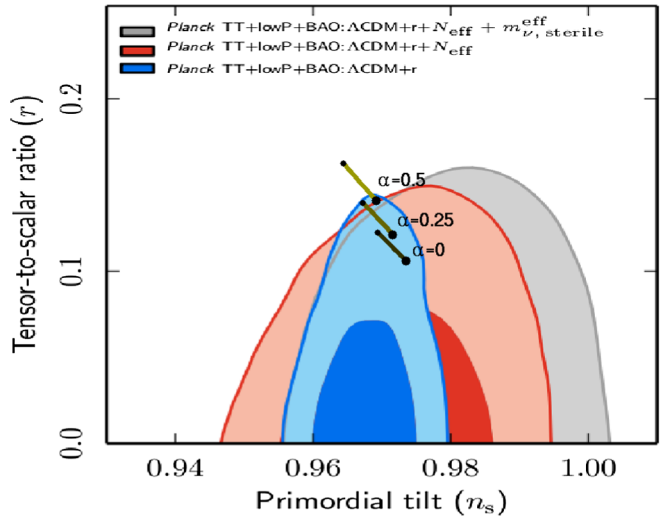

FIG. 1. Marginalized joint confidence contours for (n_s, r), at the 68% and 95% C.L., without the presence of running of the spectral indices. For the values $\alpha = 0, \frac{1}{4}$ and $\frac{1}{2}$ we have drawn the curves from 65 (small circle) to 75 (big circle) e-folds (figure courtesy of the Planck 2015 collaboration).

$$\mathcal{P} \cong \frac{H^2}{8\pi^2\epsilon_* M_{\rm pl}^2} \cong 2 \times 10^{-9}. \tag{25}$$

Using $H_* = \frac{H_E}{(\frac{\epsilon_*}{3})^{\frac{1}{2-\alpha}}}$ and $\epsilon_* = \frac{1-n_s}{4-\alpha}$, one obtains

$$H_E \sim 7 \times 10^{-4}\left(\frac{1-n_s}{3(4-\alpha)}\right)^{\frac{4-\alpha}{2(2-\alpha)}} M_{\rm pl}. \tag{26}$$

Taking, as usual, $n_s \cong 0.96$ one has the value of H_E for each value of the parameter α. In particular, for $\alpha = 0$, one has $H_E \sim 2 \times 10^{-6} M_{\rm pl} \sim 5 \times 10^{12}$ GeV, and for $\alpha = 1$, one has $H_E \sim 10^{-7} M_{\rm pl} \sim 2 \times 10^{11}$ GeV.

V. THE REHEATING PROCESS

We devote this section on the production of heavy massive particles $(m \gg H_E)$ which are conformally coupled to gravity due to a phase transition to a deflationary regime [1,9]. In this case, since the second derivative of the Hubble parameter is discontinuous, during the adiabatic regimes, we will use the first order WBK solution to define the approximate vacuum modes [10]

$$\chi_{1,k}^{WKB}(\tau) \equiv \sqrt{\frac{1}{2W_{1,k}(\tau)}} e^{-i\int^\tau W_{1,k}(\eta)d\eta}, \tag{27}$$

where $W_{1,k}$ can be calculated as

$$W_{1,k} = \omega_k - \frac{1}{4}\frac{\omega_k''}{\omega_k^2} + \frac{3}{8}\frac{(\omega_k')^2}{\omega_k^3}. \tag{28}$$

Now, before the phase transition the vacuum is depicted approximately by $\chi_{1,k}^{WKB}(\tau)$, but after the phase transition this mode becomes a mixture of positive and negative frequencies of the form $\alpha_k\chi_{1,k}^{WKB}(\tau) + \beta_k(\chi_{1,k}^{WKB})^*(\tau)$.

The β_k-Bogoliubov coefficient could be obtained, as usual, matching both expressions at the transition time τ_E, obtaining

$$\beta_k = \frac{\mathcal{W}[\chi_{1,k}^{WKB}(\tau_E^-), \chi_{1,k}^{WKB}(\tau_E^+)]}{\mathcal{W}[(\chi_{1,k}^{WKB})^*(\tau_E^+), \chi_{1,k}^{WKB}(\tau_E^+)]},$$

where $\mathcal{W}$ is the Wronskian.

The square modulus of the β_k-Bogoliubov coefficient will be given by

$$|\beta_k|^2 \cong \frac{m^4 a_E^{10}(\ddot{H}_E^+ - \ddot{H}_E^-)^2}{256(k^2 + m^2 a_E^2)^5} = \frac{81(2-\alpha)^2 m^4 a_E^{10} H_E^6}{256(k^2 + m^2 a_E^2)^5}.$$

The number and energy density are given by

$$n_\chi \equiv \frac{1}{2\pi^2 a^3}\int_0^\infty k^2|\beta_k|^2 dk, \quad \rho_\chi \equiv \frac{1}{2\pi^2 a^4}\int_0^\infty k^2\omega_k|\beta_k|^2 dk. \tag{29}$$

Then for our family one has

$$n_\chi \sim 3\times 10^{-3}(2-\alpha)^2\frac{H_E^6}{m^3}\left(\frac{a_E}{a}\right)^3, \qquad \rho_\chi \sim m n_\chi. \tag{30}$$

We notice that at the beginning of reheating, the particles are far from being in thermal equilibrium, and at first their energy density scales as a^{-3}, eventually they will decay into lighter particles, which will interact through multiple scattering. At the end of this process, the universe becomes filled with a relativistic plasma in thermal equilibrium whose energy density decays as a^{-4}. Now, since the energy density of the background decays as a^{-4} (i.e. deflationary regime), eventually the energy density of the relativistic plasma will dominate and the universe will become reheated.

Here, as in [11,12], we consider the thermalization process, where the cross section for $2 \to 3$ scattering with gauge bosons exchange whose typical energy is $\rho_\chi^{\frac{1}{4}}(0)$, is given by $\sigma = \beta^3\rho_\chi^{-\frac{1}{2}}(0)$, with $\beta^2 \sim 10^{-3}$. The thermalization rate is

$$\Gamma = \sigma n_\chi(0) \sim 5\times 10^{-2}(2-\alpha)\beta^3\left(\frac{H_E}{m}\right)^2 H_E.$$

Thermal equilibrium is reached when $\Gamma \sim H(t_{\rm eq}) \cong H_E(\frac{a_E}{a_{\rm eq}})^3$, which leads to the relation $\frac{a_E}{a_{\rm eq}} \sim 4\times 10^{-1}(2-\alpha)^{1/3}\beta(\frac{H_E}{m})^{2/3}$. Then, at the equilibrium one has

$$\rho_\chi(t_{\rm eq}) \sim 10^{-4}(2-\alpha)^3\beta^3\left(\frac{H_E}{m}\right)^4 H_E^4,$$
$$\rho(t_{\rm eq}) \sim 7\times 10^{-3}(2-\alpha)^2\beta^6\left(\frac{H_E}{m}\right)^4 H_E^2 M_{\rm pl}^2. \tag{31}$$

After this thermalization, the relativistic plasma and the background evolve as

$$\rho_\chi(t) = \rho_\chi(t_{\rm eq})\left(\frac{a_{\rm eq}}{a}\right)^4, \qquad \rho(t) = \rho(t_{\rm eq})\left(\frac{a_{\rm eq}}{a}\right)^6, \tag{32}$$

and the reheating is obtained when both energy densities are of the same order, which happens when $\frac{a_{\rm eq}}{a_R} \sim \sqrt{\frac{\rho_\chi(t_{\rm eq})}{\rho(t_{\rm eq})}}$, and thus, one obtains a reheating temperature of the order

$$T_R \sim \rho_\chi^{\frac{1}{4}}(t_{\rm eq})\sqrt{\frac{\rho_\chi(t_{\rm eq})}{\rho(t_{\rm eq})}} \sim 10^{-1}\left(\frac{H_E}{M_{\rm pl}}\right)^2\left(\frac{H_E}{m}\right)M_{\rm pl}.$$

Since $H_E \ll m$, if we consider masses of the order $10^2 H_E$ one has

$$T_R \sim 10^{-3}\left(\frac{H_E}{M_{\rm pl}}\right)^2 M_{\rm pl} \sim 5\times 10^{-10}\left(\frac{1-n_s}{3(4-\alpha)}\right)^{\frac{4-\alpha}{2-\alpha}} M_{\rm pl}.$$

As particular cases we consider

1. The quadratic potential corresponding to $\alpha = 0$ leads to the reheating temperature $T_R \sim 5\times 10^{-15} M_{\rm pl} \sim 10^4$ GeV.
2. The cubic potential corresponding to $\alpha = \frac{2}{3}$ leads to the reheating temperature $T_R \sim 5\times 10^{-16} M_{\rm pl} \sim 10^3$ GeV.
3. The quartic potential corresponding to $\alpha = 1$ leads to the reheating temperature $T_R \sim 4\times 10^{-17} M_{\rm pl} \sim 10^2$ GeV.

Finally, to end this section, we study the evolution after reheating. Since after the phase transition the potential is constant one will have

$$\ddot{\varphi} + 3H\dot{\varphi} = 0 \Leftrightarrow \dot{\varphi}(t) = \dot{\varphi}(t_R)e^{-3\int_{t_R}^{t} H(s)ds}, \tag{33}$$

where t_R is the reheating time.

On the other hand, during the radiation and the matter dominated phases, one will have

$$H(t) = \frac{H_R}{1+2(t-t_R)H_R}, \text{ and } H(t) = \frac{2H_M}{2+3(t-t_M)H_M}, \tag{34}$$

where the subindices R, M respectively denote the Hubble rate when radiation and matter domination will start to dominate. Then if we denote by t_Λ the time when the cosmological constant starts to dominate on, one will get

$$\dot{\varphi}(t_\Lambda) = \frac{\dot{\varphi}(t_R)}{(1+2(t_M-t_R)H_R)^{\frac{3}{2}}(2+3(t_\Lambda-t_M)H_M)^2}. \quad (35)$$

Since nowadays the universe is accelerating one can take $\Lambda \sim H_0^2$, where H_0 is the current value of the Hubble parameter, and thus, one arrives at

$$\dot{\varphi}^2(t_\Lambda) \sim \dot{\varphi}^2(t_R)\frac{H_M H_0^2}{H_R^3}. \quad (36)$$

As a consequence, since at the beginning of the radiation domination, all the energy density is kinetic, the ratio between the kinetic and potential energy density ($\mathcal{R}$) when the cosmological constant starts to dominate satisfies

$$\mathcal{R} \cong \frac{\dot{\varphi}^2(t_\Lambda)/2}{\Lambda M_{\rm pl}^2} \sim \frac{H_M}{H_R}. \quad (37)$$

Now using that the value of the Hubble parameter at the beginning of the matter domination is of the order $H_M \sim 10^{-54} M_{\rm pl}$ (see [9]) and for our models H_R belongs between $10^{-30} M_{\rm pl}$ and $10^{-34} M_{\rm pl}$, one can calculate that

$$\mathcal{R} \leq 10^{-20}, \quad (38)$$

which means that the kinetic part of the energy density is subdominant, and thus, in our model, it is the cosmological constant which drives the current evolution of the universe.

VI. CALCULATION OF THE NUMBER OF *e*-FOLDS

We start with the main formula [7]

$$\frac{k_*}{a_0 H_0} = e^{-N_*}\frac{H_*}{H_0}\frac{a_{\rm end}}{a_E}\frac{a_E}{a_R}\frac{a_R}{a_M}\frac{a_M}{a_0} = e^{-N_*}\frac{H_*}{H_0}\frac{a_{\rm end}}{a_E}\frac{\rho_R^{-1/12}\rho_M^{1/4}}{\rho_E^{1/6}}\frac{a_M}{a_0}, \quad (39)$$

where "end," R and M respectively symbolize the end of inflation, the beginning of radiation era, and the beginning of the matter domination era. Further, the subindex "0" at any quantity means its value at current time. Here we have used the relations

$$\left(\frac{a_E}{a_R}\right)^6 = \frac{\rho_R}{\rho_E}, \qquad \left(\frac{a_R}{a_M}\right)^4 = \frac{\rho_M}{\rho_R}. \quad (40)$$

Taking the pivot scale as $k_* = 0.05\ {\rm Mpc}^{-1}$, and since the current horizon scale is $a_0 H_0 \cong 2\times 10^{-4}\ {\rm Mpc}^{-1}$, one obtains

$$N_* = -5.52 + \ln\left(\frac{H_*}{H_0}\right) + \ln\left(\frac{a_{\rm end}}{a_E}\right) + \frac{1}{4}\ln\left(\frac{\rho_M}{\rho_R}\right) + \frac{1}{6}\ln\left(\frac{\rho_R}{\rho_E}\right) + \ln\left(\frac{a_M}{a_0}\right). \quad (41)$$

Since after reheating, the process becomes adiabatic, i.e. $T_0 = \frac{a_M}{a_0}T_M$, hence using the relations $\rho_M \cong \frac{\pi^2}{15} g_M T_M^4$ and $\rho_R \cong \frac{\pi^2}{30} g_R T_R^4$ (where g_i's, $i = R$, M are the relativistic degrees of freedom[1] [13]), one arrives at

$$N_* = -5.52 + \ln\left(\frac{H_*}{H_0}\right) + \ln\left(\frac{a_{\rm end}}{a_E}\right) + \frac{1}{4}\ln\left(\frac{2g_M}{g_R}\right) + \frac{1}{6}\ln\left(\frac{\rho_R}{\rho_E}\right) + \ln\left(\frac{T_0}{T_R}\right). \quad (42)$$

Now, taking into account that $H_0 \sim 6\times 10^{-61} M_{\rm pl}$ and $\mathcal{P} = \frac{H_*^2}{8\pi^2\epsilon_* M_{\rm pl}^2} \sim 2\times 10^{-9}$ one obtains

$$\ln\left(\frac{H_*}{H_0}\right) = 131.38 + \frac{1}{2}\ln\left(\frac{1-n_s}{3(4-\alpha)}\right). \quad (43)$$

Now using the current temperature of the universe $T_0 \cong 2.73\ {\rm K} \cong 2\times 10^{-13}\ {\rm GeV}$ and $g_M = 3.36$ [13] one has

$$\frac{1}{4}\ln\left(\frac{2g_M}{g_R}\right) + \ln\left(\frac{T_0}{T_R}\right) = -28.76 - \ln\left(\frac{g_R^{\frac{1}{4}}T_R}{\rm GeV}\right). \quad (44)$$

From the value of the Hubble parameter at the transition time, one will obtain

$$\frac{1}{6}\ln\left(\frac{\rho_R}{\rho_E}\right) = -26.16 - \frac{4-\alpha}{6(2-\alpha)}\ln\left(\frac{1-n_s}{3(4-\alpha)}\right) + \frac{2}{3}\ln\left(\frac{g_R^{\frac{1}{4}}T_R}{\rm GeV}\right). \quad (45)$$

Collecting all the terms one obtains

$$N_* = 70.94 + \ln\left(\frac{a_{\rm end}}{a_E}\right) + \frac{1-\alpha}{3(2-\alpha)}\ln\left(\frac{1-n_s}{3(4-\alpha)}\right) - \frac{1}{3}\ln\left(\frac{g_R^{\frac{1}{4}}T_R}{\rm GeV}\right). \quad (46)$$

[1]Specifically g_M stands for the number of relativistic degrees of freedom at matter radiation equality and g_R is the number of relativistic degrees of freedom at the end of reheating.

On the other hand, a simple calculation leads to

$$\ln\left(\frac{a_{\rm end}}{a_E}\right) = \int_{H_E}^{H_{\rm end}} \frac{H}{\dot{H}} dH = -\frac{2}{3(2-\alpha)}, \tag{47}$$

$$N_* = 70.94 - \frac{1}{3(2-\alpha)}\left[2-(1-\alpha)\ln\left(\frac{1-n_s}{3(4-\alpha)}\right)\right] - \frac{1}{3}\ln\left(\frac{g_R^{\frac{1}{4}} T_R}{\rm GeV}\right). \tag{48}$$

Finally, since for our models $T_R \sim 5\times 10^{-10}(\frac{1-n_s}{3(4-\alpha)})^{\frac{4-\alpha}{2-\alpha}} M_{\rm pl} \sim 10^8 (\frac{1-n_s}{3(4-\alpha)})^{\frac{4-\alpha}{2-\alpha}}$ GeV, hence using the fact that $g_R = 107$ for $T_R \geq 175$ GeV [13], one gets

$$N_* = 64.41 - \frac{1}{3(2-\alpha)}\left[2 + 3\ln\left(\frac{1-n_s}{3(4-\alpha)}\right)\right]. \tag{49}$$

Taking as usual $n_s \cong 0.96$ we observe the following:

1. For the quadratic potential ($\alpha = 0$), the number of e-folds is $N_* = 67$.
2. For the cubic potential ($\alpha = \frac{2}{3}$), the number of e-folds is $N_* = 68$.
3. For the quartic potential ($\alpha = 1$), the number of e-folds is $N_* = 69$.

VII. COMPARISON WITH THE PREVIOUS MODEL

In Sec. VII of our previous work [1], we introduced the following dynamical system:

$$\dot{H} = \begin{cases} -3H_e^2(2H - H_e) & \text{for } H > H_E \\ -3H^2 + \Lambda & \text{for } H \leq H_E, \end{cases} \tag{50}$$

where H_e is the model parameter and the phase transition occurs at $H_E = H_e + \sqrt{\frac{\Lambda}{3}}$. We note that this background originally comes from the following quintessential Higgs-style potential [1,14]:

$$V(\varphi) = \begin{cases} \frac{27H_e^2 M_{\rm pl}^2}{16}\left(\frac{\varphi^2}{M_{\rm pl}^2} - \frac{2}{3}\right)^2 & \text{for } \varphi < \varphi_E \\ \Lambda M_{\rm pl}^2 & \text{for } \varphi \geq \varphi_E, \end{cases} \tag{51}$$

in which $\varphi_E = -M_{\rm pl}\sqrt{\frac{2}{3}}\sqrt{1 + \frac{2}{H_e}\sqrt{\frac{\Lambda}{3}}} \cong -M_{\rm pl}\sqrt{\frac{2}{3}}$.

It has been shown in [1,9] that due to the gravitational production of heavy massive particles, the reheating temperature belongs in the MeV regime. This is due to the fact that the second derivative of the Hubble parameter is continuous and the third one is discontinuous at the transition time. However, for the family of models described by the sole parameter $\alpha \in [0, 1]$, the second derivative of the Hubble parameter is discontinuous at the transition phase which leads to a reheating temperature in the GeV or TeV regime.

Moreover, during the inflationary period, the Higgs-style potential has the same behavior as a quartic one (i.e. the model with $\alpha = 1$), which means that it does not match with Planck 2015 data (see Fig. 1). On the contrary, for our new family of models, if one takes $\alpha \in [0, \frac{1}{2})$, the corresponding models match at 2σ C.L. with Planck 2013 and Planck 2015 without allowing the running. It is one of the main results of the present work which proves the potentiality of the current family of models compared to our earlier work [1].

VIII. SUMMARY AND DISCUSSION

The current work offers a sequel of our previous work [1] with a significant improvement compared to its mathematical simplicity, and in agreement with the very latest observational data. Let us demonstrate the main improvements by comparing the previous model [1] with the current one and finally the need of this sequel.

In [1] a simple unified cosmological model was proposed having (i) an inflationary period described by a Higgs type potential, (ii) a sudden phase transition from the inflationary phase to the deflationary phase, which results in the production of massive particles, hence the universe begins to reheat, after that, it successively enters into the radiation and matter dominated eras, and finally, (iii) a quintessential stage explained by an exponential potential. Additionally, although the model was geodesically past incomplete, but it did not encounter any big bang singularity in the finite cosmic time. So, essentially, we realized a singularity free cosmological model unifying the early inflationary epoch with the current cosmic acceleration by only a single scalar field whose potential is a combination of Higgs potential and an exponential one. Also, the model provided a complete analytic background which thus was helpful to calculate the associated cosmological parameters. We found that the model only agrees with the Planck 2013 data [2] in the presence of running, however, as we have already discussed, the model does not match with the Planck's observational data without the presence of running.

But in the present work we provide an improved version of [1] which is potential and worthy for further discussions. Here we propose a family of new cosmological models described by a sole parameter $\alpha \in [0, 1]$ which provides a complete picture of our universe via a single scalar field as in [1]. The current family of models has (i) an inflationary phase described by a power law potential, (ii) a sudden phase transition from inflationary regime to the deflationary regime, hence beginning of reheating, consequently, successive radiation, matter dominated eras, and finally (iii) the models enter into the current accelerating phase responsible by the cosmological constant. In addition to that, the family of models is nonsingular in nature,

i.e. they do not predict any finite cosmic time big bang singularity, but are geodesically past incomplete. That means similar to [1] the present family of models also unifies the inflationary epoch with the current accelerating phase by a combination of a power law potential and a cosmological constant. In particular, the inflationary power law potentials are recognized by the models with $\alpha = 0, \frac{2}{3}$, 1, as quadratic, cubic and quartic potentials in which for the observable modes, the universe inflates respectively for a number of 67, 68, and 69 e-folds. Another interesting point in the family of models is that the reheating temperature could reach the GeV or TeV regime depending on the value of the sole parameter "α" unlike in the previous model [1] where the reheating temperature belongs in the MeV regime. Finally, we found that a large number of models having $\alpha \in [0, \frac{1}{2})$ match both Planck 2013 and Planck 2015 data at 2σ C.L. without the need of running, which does not happen with the model presented in [1].

Summarizing, the current family of models describes a simple nonsingular inflationary quintessential analytic cosmological models, providing a greater reheating temperature in the GeV/TeV regime, and a large number of models belonging to this family are in excellent agreement with the Planck 2015 data without allowing the running, and hence it reports a significant improvement of [1].

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