1) Calculem la transformada de Laplace de:

(a) $f(t) = \begin{cases} 2t+1, & \text{si } 0 < t < 1 \\ 0, & \text{si } t \geq 1 \end{cases}$

$$F(s) = \int_0^{+\infty} e^{-st} f(t)\,dt = \int_0^1 e^{-st}\cdot(2t+1)\,dt + \underbrace{\int_1^{+\infty} e^{-st}\cdot 0\,dt}_{=0} =$$

$$= \left\{ \begin{array}{l} u = 2t+1 \to du = 2dt \\ dv = e^{-st}dt \to v = -e^{-st}/s \end{array} \right\} = \left[(2t+1)\frac{e^{-st}}{-s} \right]_{t=0}^{t=1} - \int_0^1 \frac{e^{-st}}{-s}\cdot 2\,dt =$$

$$= -\frac{3e^{-s}}{s} + \frac{1}{s} + \left[-2\cdot\frac{e^{-st}}{s^2} \right]_{t=0}^{t=1} = -\frac{3e^{-s}}{s} + \frac{1}{s} - \frac{2e^{-s}}{s^2} + \frac{2}{s^2}$$

(b) $f(t) = t\cdot \sin t$

$$F(s) = \mathcal{L}[t \sin t] = -\frac{d}{ds}\mathcal{L}[\sin t] = -\frac{d}{ds}\frac{1}{s^2+1} = \frac{2s}{(s^2+1)^2},$$

(c) $f(t) = (2t-1)^3$

$$F(s) = \mathcal{L}[(2t-1)^3] = \mathcal{L}[8t^3 - 12t^2 + 6t - 1] = \frac{48}{s^4} - \frac{24}{s^3} + \frac{6}{s^2} - \frac{1}{s}.$$

(d) $f(t) = \cos^2 t$

$$F(s) = \mathcal{L}\left[\frac{1+\cos 2t}{2}\right] = \frac{1}{2}\left(\frac{1}{s} + \frac{s}{s^2+4}\right) = \frac{s^2+2}{s(s^2+4)}$$

(e) $f(t) = \sin t \cdot \sin 2t$

$$F(s) = \mathcal{L}\left[\frac{1}{2}(\cos t - \cos 3t)\right] = \frac{1}{2}\left(\frac{s}{s^2+1} - \frac{s}{s^2+9}\right) = \frac{4s}{s^4+10s^2+9}$$

↑ usem: $\sin\alpha \cdot \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

(f) $f(t) = e^t \sinh(t)$

$$F(s) = \mathcal{L}[e^t \cdot \sinh(t)] = \mathcal{L}[\sinh(t)]_{s\to s-1} = \frac{1}{(s-1)^2-1}$$