

7) Calculen la transformada de Laplace

(a)  $f(t) = e^{2-t} u(t-2)$

$$F(s) = \mathcal{L}[e^{2-t} u(t-2)] = \mathcal{L}[e^{-(t-2)} u(t-2)] = e^{-2s} \mathcal{L}[e^{-t}] = \frac{e^{-2s}}{s+1}$$

(b)  $f(t) = e^t \cos^2 3t$

$$F(s) = \mathcal{L}[e^t \cos^2 3t] = \mathcal{L}[\cos^2 3t]_{s \rightarrow s-1} = \mathcal{L}\left[\frac{1 + \cos 6t}{2}\right]_{s \rightarrow s-1}$$

$$= \frac{1}{2} \left( \frac{1}{s-1} + \frac{s-1}{(s-1)^2 + 36} \right)$$

(c)  $f(t) = t e^{-3t} \cos 3t$

$$F(s) = \mathcal{L}[t e^{-3t} \cos 3t] = -\frac{d}{ds} \mathcal{L}[\cos 3t]_{s \rightarrow s+3} = -\frac{d}{ds} \left[ \frac{s}{s^2+9} \right]_{s \rightarrow s+3}$$

$$= -\frac{(s^2+9) - s \cdot (2s)}{(s^2+9)^2} \Big|_{s \rightarrow s+3} = \frac{-9 + (s+3)^2}{[(s+3)^2+9]^2}$$

(d)  $f(t) = t \int_0^t z e^{-z} dz$

$$F(s) = \mathcal{L}[f(t)] = -\frac{d}{ds} \mathcal{L}\left[\int_0^t z e^{-z} dz\right] = -\frac{d}{ds} \left( \frac{1}{s} \mathcal{L}[t e^{-t}] \right) =$$

$$= +\frac{d}{ds} \left( \frac{1}{s} \frac{d}{ds} \mathcal{L}[e^{-t}] \right) = \frac{d}{ds} \left( \frac{1}{s} \frac{d}{ds} \left( \frac{1}{s+1} \right) \right) = \frac{d}{ds} \left( -\frac{1}{s(s+1)^2} \right) =$$

$$= \frac{1}{s(s+1)} \left( \frac{1}{s} + \frac{2}{s+1} \right)$$

(e)  $f(t) = e^{2t} * \sin t = \int_0^t e^{2z} \sin(t-z) dz$  (Convolution)

$$F(s) = \mathcal{L}[e^{2t} * \sin t] = \mathcal{L}[e^{2t}] \cdot \mathcal{L}[\sin t] = \frac{1}{(s-2)(s^2+1)}$$