(3) Calculeu una antitransformada de Laplace:

(a) $g(s) = \dfrac{(s+2)^2}{s^3}$

$$\mathcal{L}^{-1}\left[\frac{(s+2)^2}{s^3}\right] = \mathcal{L}^{-1}\left[\frac{1}{s} + \frac{4}{s^2} + \frac{4}{s^3}\right] = 1 + 4t + 2t^2.$$

(b) $g(s) = \dfrac{1}{5 \cdot s - 2}$

$$\mathcal{L}^{-1}\left[\frac{1}{5\cdot s - 2}\right] = \frac{1}{5}\mathcal{L}^{-1}\left[\frac{1}{s - 2/5}\right] = \frac{e^{2/5\,t}}{5}\,\mathcal{L}^{-1}\left[\frac{1}{s}\right] = \frac{e^{2/5\,t}}{5}$$

(c) $g(s) = \dfrac{s+1}{(s^2-4s)(s+5)}$

Descomposicus en fraccions simples:

$$\frac{s+1}{s(s-4)(s+5)} = \frac{A}{s} + \frac{B}{s-4} + \frac{C}{s+5} \iff s+1 = A(s-4)(s+5) + B\,s\,(s+5) + C\cdot s\,(s-4)$$

Fent $s=0$, $s=4$ i $s=5$ obtenim $A = -\frac{1}{20}$, $B = 5/36$, $C = -4/45$

$$\mathcal{L}^{-1}[g(s)] = \mathcal{L}^{-1}\left[\frac{-1/20}{s}\right] + \mathcal{L}^{-1}\left[\frac{5/36}{s-4}\right] + \mathcal{L}^{-1}\left[\frac{-4/45}{s+5}\right] =$$

$$= -\frac{1}{20} + \frac{5}{36}e^{+4t} - \frac{4}{15}e^{-5t}.$$

(d) $g(s) = \dfrac{1}{s^4 - 9}$

Fem $\dfrac{1}{s^4-9} = \dfrac{A}{s^2-3} + \dfrac{B}{s^2+3} \iff 1 = A(s^2+3) + B(s^2-3)$ d'on $A = 1/6$, $B = -1/6$

$$\mathcal{L}^{-1}[g(s)] = \mathcal{L}^{-1}\left\{\frac{1/6}{s^2-3}\right\} + \mathcal{L}^{-1}\left\{\frac{-1/6}{s^2+3}\right\} = \frac{1}{6\sqrt{3}}\sinh(\sqrt{3}t) - \frac{1}{6\sqrt{3}}\sin(\sqrt{3}t)$$

(De fet la descomposicus en fraccus simples és:

$$\frac{1}{s^4-9} = \frac{A}{s-\sqrt{3}} + \frac{B}{s+\sqrt{3}} + \frac{C+Ds}{s^2+3}\,).$$