(8) Calcular una antitransformada de

(a) $g(s) = \dfrac{2s+5}{s^2+6s+34}$

Descomposició en fraccions simples

$$\underbrace{\frac{2s+5}{s^2+6s+34}}_{\text{denominador no té arrels reals}} = \frac{2s+5}{(s+3)^2+25} = \frac{2(s+3)}{(s+3)^2+25} - \frac{1}{(s+3)^2+25}$$

$$\mathcal{L}^{-1}[g(s)] = 2\mathcal{L}^{-1}\left[\frac{s+3}{(s+3)^2+5^2}\right] - \mathcal{L}^{-1}\left[\frac{1}{(s+3)^2+5^2}\right] =$$

$$= e^{-3t}\left(2\mathcal{L}^{-1}\left[\frac{s}{s^2+5^2}\right] - \frac{1}{5}\mathcal{L}^{-1}\left[\frac{5}{s^2+5^2}\right]\right) = e^{-3t}\left(2\cos 5t - \frac{1}{5}\sin 5t\right)$$

(b) $g(s) = \dfrac{e^{-2s}}{s^2(s-1)}$

Descomposició en fraccions simples: $\dfrac{1}{s^2(s-1)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s-1} \Rightarrow$

$\Rightarrow 1 = As(s-1) + B(s-1) + Cs^2 \Rightarrow C = 1,\ B = -1,\ A = -1$

$$\mathcal{L}^{-1}[g(s)] = \left(\mathcal{L}^{-1}\left[\frac{1}{s^2(s-1)}\right]\right)_{t \to t-2} \cdot u(t-2) = \mathcal{L}^{-1}\left[-\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s-1}\right]_{t\to t-2} \cdot u(t-2) =$$

$$= (-1 - t + e^{+t})_{t \to t-2}\, u(t-2) = (-1 - (t-2) + e^{t-2})\, u(t-2) =$$

$$= (1 - t + e^{t-2}) \cdot u(t-2)$$