(11) Calculem una antitransformada de Laplace de

$$g(s) = \ln\left(\frac{s^2+1}{s^2+4}\right) \quad \text{usant} \quad \mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \mathcal{L}[f(t)](s)$$

Fem $n=1$ i $f(t) = \mathcal{L}^{-1}[g(s)]$. Llavors:

$$\mathcal{L}[t f(t)] = -\frac{d}{ds} g(s) = -\frac{d}{ds} \ln\left(\frac{s^2+1}{s^2+4}\right) = -\frac{d}{ds}\left[\ln(s^2+1) - \ln(s^2+4)\right] =$$

$$= -\frac{2s}{s^2+1} + \frac{2s}{s^2+4}$$

Així: $$t f(t) = -2\mathcal{L}^{-1}\left[\frac{s}{s^2+1}\right] + 2\mathcal{L}^{-1}\left[\frac{s}{s^2+4}\right] =$$

$$= -2\cos t + 2\cos(2t)$$

$$f(t) = \frac{2}{t}(\cos 2t - \cos t)$$