17) Sigui $f(t) = \begin{cases} 2t, & 0 \le t \le 1 \\ t, & t > 1 \end{cases} = 2t - t \cdot U(t-1)$

(a) Calculeu $\mathcal{L}[f(t)]$.

(i) Per la definició de $f(t)$ i de $\mathcal{L}$

$$F(s) = \mathcal{L}[f(t)] = \int_0^1 e^{-st} \cdot 2t \, ds + \int_1^{\infty} e^{-st} \cdot t \, dt = \left\{ \begin{array}{l} u = t \to du = dt \\ dv = e^{-st} dt \to v = -\frac{e^{-st}}{s} \end{array} \right\} =$$

$$= 2\left(\left[-\frac{te^{-st}}{s} \right]_{t=0}^{t=1} + \int_0^1 \frac{e^{-st}}{s} dt \right) + \left(\left[-\frac{te^{-st}}{s} \right]_{t=1}^{t=\infty} + \int_1^{\infty} \frac{e^{-st}}{s} dt \right) =$$

$$= -2\frac{e^{-s}}{s} - 2\left[\frac{e^{-st}}{s^2} \right]_{t=0}^{t=1} + \frac{e^{-s}}{s} - \left[\frac{e^{-st}}{s^2} \right]_{t=1}^{t=\infty} = -\frac{2e^{-s}}{s} - \frac{2e^{-s}}{s^2} + \frac{2}{s^2} + \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} =$$

$$= -\frac{e^{-s}}{s} + \frac{2}{s^2} - \frac{e^{-s}}{s^2}$$

(ii) Via l'expressió de $f(t)$ en termes de $U(t-1)$

$$F(s) = \mathcal{L}[2t] - \mathcal{L}[t \cdot U(t-1)] = \frac{2}{s^2} - \mathcal{L}[(t-1)U(t-1) + U(t-1)] =$$

$$= \frac{2}{s^2} - e^{-s}\mathcal{L}[t] - e^{-s}\mathcal{L}[1] = \frac{2}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$

(b) Calculeu $\mathcal{L}[f'(t)]$

$$f'(t) = \begin{cases} 2, & 0 < t < 1 \\ 1, & t > 1 \end{cases} = 2 - U(t-1)$$

$$G(s) = \mathcal{L}[f'(t)] = \mathcal{L}[2 - U(t-1)] = \frac{2}{s} - \frac{e^{-s}}{s}$$

(c) Es compleix la fórmula $\underbrace{\mathcal{L}[f'(t)](s)}_{= G(s) = \frac{2}{s} - \frac{e^{-s}}{s}} = s\underbrace{\mathcal{L}[f(t)](s)}_{= F(s) = -\frac{e^{-s}}{s} + \frac{2}{s^2} - \frac{e^{-s}}{s^2}} - \underbrace{f(0)}_{= 0}$? No

Què falla?

↑

f no és contínua en $t = 1$