(24) Resolver $y'' + 4y' + 13y = \delta(t-\pi) + \delta(t-3\pi)$, $y(0)=1, y'(0)=0$

$$Y(s) = \mathcal{L}\{y(t)\}$$

$$s^2 Y(s) - s\underbrace{y(0)}_{=1} - s\underbrace{y'(0)}_{=0} + 4[sY(s) - \underbrace{y(0)}_{=1}] + 13Y(s) = e^{-\pi s} + e^{-3\pi s}$$

$$(s^2+4s+13)Y(s) = e^{-\pi s} + e^{-3\pi s} + s + 4$$

$$Y(s) = \frac{e^{2\pi}\cdot e^{-\pi(s+2)} + e^{6\pi}\cdot e^{-3\pi(s+2)}}{(s+2)^2+3^2} + \frac{s+2}{(s+2)^2+3^2} + \frac{2}{(s+2)^2+3^2}$$

$$y(t) = e^{2\pi}\mathcal{L}^{-1}\left\{\frac{e^{-\pi(s+2)}}{(s+2)^2+3^2}\right\} + e^{6\pi}\mathcal{L}^{-1}\left\{\frac{e^{-3\pi(s+2)}}{(s+2)^2+3^2}\right\} +$$

$$+\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+3^2}\right\} + \frac{2}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s+2)^2+3^2}\right\} =$$

$$= e^{2\pi}e^{-2t}\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+3^2}\right\} + e^{6\pi}e^{-2t}\mathcal{L}^{-1}\left\{\frac{e^{-3\pi s}}{s^2+3^2}\right\} +$$

$$+ e^{-2t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\} + \frac{2}{3}e^{-2t}\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\} =$$

$$= \frac{1}{3}e^{2\pi}e^{-2t}\mathcal{L}\left\{\frac{3}{s^2+3}\right\}_{t\to t-\pi} u(t-\pi) + \frac{1}{3}e^{6\pi}e^{-2t}\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\}_{t\to t-3\pi} u(t-3\pi) +$$

$$+ e^{-2t}\cos(3t) + \frac{2}{3}e^{-2t}\sin(3t) =$$

$$= \frac{1}{3}e^{2\pi}e^{-2t}\sin(3(t-\pi))u(t-\pi) + \frac{1}{3}e^{6\pi}e^{-2t}\sin(3(t-3\pi))u(t-3\pi) +$$

$$+ e^{-2t}\cos(3t) + \frac{2}{3}e^{-2t}\sin(3t) =$$

$$= -\frac{1}{3}e^{2(\pi-t)}\sin(3t)u(t-\pi) - \frac{1}{3}e^{2(3\pi-t)}\sin(3t)u(t-3\pi) + e^{-2t}\cos(3t) +$$

$$+ \frac{2}{3}e^{-2t}\sin(3t)$$