(28) Resoleu l'equació integro-diferencial

$$y'(t) + \int_0^t y(t-u)\, e^{-2u}\, du = 0, \quad y(0) = 1$$

Fem $Y(s) = \mathcal{L}[y(t)](s)$ i apliquem transformada de Laplace a l'equació, tot expressant l'integral com un producte de convolució:

$$y'(t) + y(t) * e^{-2t} = 0$$

$$\mathcal{L}[y'(t)] + \mathcal{L}[y(t) * e^{-2t}] = 0$$

$$sY(s) - y(0) + \mathcal{L}[y(t)] \cdot \mathcal{L}[e^{-2t}] = 0$$

$$sY(s) - 1 + Y(s)\,\frac{1}{s+2} = 0$$

$$\frac{(s^2+2s+1)}{s+2}\, Y(s) = 1$$

$$Y(s) = \frac{s+2}{(s+1)^2} = \frac{1}{s+1} + \frac{1}{(s+1)^2}$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] + \mathcal{L}^{-1}\left[\frac{1}{(s+1)^2}\right] = e^{-t}\left(\mathcal{L}^{-1}\left[\frac{1}{s}\right] + \mathcal{L}^{-1}\left[\frac{1}{s^2}\right]\right) =$$

$$= e^{-t}(1+t)$$