BERLIN-POZNAŃ-HAMBURG SEMINAR
20TH ANNIVERSARY
29-30 MAY 2015

PROGRAM

Friday 29

13:30-13:45 Arrival of participants.
13:45-14:00 Welcome remarks.

14:00-14:45 Tomasz Łuczak: *On the chromatic number of generalized shift graphs.*
14:45-15:15 Coffee Break.

15:15-15:45 Tuan Tran: *A removal lemma for large nearly-intersecting families.*
15:45-16:15 Joanna Polcyn: *Multicolor Ramsey numbers for* 3*-uniform loose paths of length* 3.
16:15-16:45 Dennis Clemens: *Rainbow matchings in multigraphs.*
16:45-17:15 Coffee Break.

17:15-18:00 Christian Reiher: *On a Turán problem in weakly quasirandom* 3*-uniform. hypergraphs.*

18:00 Group picture.

19:00- Joint dinner at Luise.

Saturday 30

09:30-10:15 Juanjo Rué: *Subgraph statistics in subcritical graph classes.*
10:15-10:45 Coffee Break.

10:45-11:15 Justyna Tabor: *k-factors in stochastic Kronecker graphs.*
11:15-11:45 Guilherme Mota: *Decompositions of highly connected graphs into paths.*
11:45-12:15 Konstanty Junosza-Szaniawski: *Circular, fractional and j-fold colorings of the plane.*

12:15-12:30 Concluding remarks.

Location

Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB)
Takustraße 7, D-14195 Berlin-Dahlem
Main lecture room

ABSTRACTS

DENNIS CLEMENS

Rainbow matchings in multigraphs.

A conjecture of Brualdi and Stein states that every Latin square of order n should contain a transversal of size $n-1$. Related to this problem, Aharoni and Berger conjectured that every family of n matchings of size $n+1$ in a bipartite multigraph contains a rainbow matching of size n. We prove that matching sizes of $3n/2+o(n)$ suffice to guarantee such a rainbow matching, improving previous results by Aharoni, Charbit and Howard, and Kotlar and Ziv. We also discuss a result when the (not necessarily bipartite) multigraph consists of n unions of cliques, giving an asymptotically tight answer to a question of Grinblat.

Joint work with Julia Ehrenmüller and Alexey Pokrovskiy.

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KONSTANTY JUNOSZA-SZANIAWSKI

Circular, fractional and j-fold colorings of the plane.

We consider the circular and the fractional version of the famous Nelson-Hadwiger problem, i.e. the problem of finding the chromatic number of the graph G_1 whose vertices are all points of the plane and edges join all pairs of points at distance equal to 1. It is known that the number is at least 4 and at most 7.

An *r-circular coloring* of a graph G is an assignment of arcs of length 1 of a circle of perimeter r to vertices of G in such a way that adjacent vertices get disjoint arcs. The *circular chromatic number* of G is the infimum over all r such that there exists an r-circular coloring of G. It is known (see [4]) that the circular chromatic number of a graph does not exceed its chromatic number, but is larger than the chromatic number minus one.

We show that the r-circular chromatic number of the graph G_1 does not exceed $4+4\sqrt{3}/3 \approx 6.3$. It is the first result that improves the upper bound of 7 for this number. The lower bound equal to 4 was proved by Devos at al. [1].

A *j-fold colouring* of a graph is an assignment of j-elemental sets of colors to its vertices in such a way that the sets assigned to any two adjacent vertices are disjoint. We construct some j-fold colorings of the plane for small values of j. In particular we show 2-fold and 3-fold colorings with 12 and 16 colors, respectively. For large values of j a j-fold coloring is closely related to a fractional coloring of the plane (see [3]).

Moreover, we generalize the above results for a graph $G_{[a,b]}$ (introduced by Exoo [2]) whose vertex set is the set of all points of the plane, and the edge set consists of all pairs of points at distance from the interval $[a, b]$.

References

[1] M. Devos, J. Ebrahimi, M. Ghebleh, L. Goddyn, B. Mohar, R. Naserasr, *Circular coloring the plane*, SIAM Journal on Discrete Mathematics, 21, 2 (2007), 461-465.

[2] G. Exoo, e-unit distance graphs, *Discrete Computational Geometry*, 33 (2005), 117-123.

[3] E.R. Scheinerman, D.H. Ullman, *Fractional Graph Theory*, John Wiley and Sons, 2008.

[4] A. Vince, Star chromatic number, *Journal of Graph Theory* 12 (1988), 551-559.

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TOMASZ ŁUCZAK

On the chromatic number of generalized shift graphs.

Let us consider a graph whose vertices are k-element subsets of a linearly ordered set, and two vertices $(a_1, ..., a_k)$ and $(b_1, ..., b_k)$ are adjacent if the sets which correspond to those vertices form certain prescribed "pattern" such as $a_1 < a_2 < b_1 < a_3 < b_2 < ... < b_{k-2} < a_k < b_{k-1} < b_k$.

In the talk we study the chromatic number of such graphs. This is a joint work with Christian Avart and Vojta Rödl.

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GUILHERME MOTA

Decompositions of highly connected graphs into paths.

We study the Decomposition Conjecture posed by Barát and Thomassen (2006), which states that for every tree T there exists a natural number $k = k(T)$ such that, if G is a k-edge-connected graph and $|E(T)|$ divides $|E(G)|$, then G admits a decomposition into copies of T. This conjecture was verified for stars, some bistars, paths whose length is a power of 2, and paths of length 3. We verify the Decomposition Conjecture for paths of any fixed length. In this talk we will sketch the proof of the Decomposition Conjecture for paths of length 5.

Joint work with F. Botler, M.T.I. Oshiro and Y. Wakabayashi.

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JOANNA POLCYN

Multicolor Ramsey numbers for 3-uniform loose paths of length 3.

Let P stand for a 3-uniform loose path of length three. Given an integer $r \geq 2$, let $R(P;r)$ be the smallest integer n such that every r-coloring of the $\binom{n}{3}$ triples from $\{1, \ldots, n\}$ results in a monochromatic copy of P. It is known that $R(P;r) \geq r+6$ and $R(P;r) = r+6$ for $r = 2, 3$. By a subtle analysis of the Turán numbers for P, I will show that $R(P;r) = r+6$ also for $r = 4, 5, 6, 7$. Along the way, some interesting "restricted" Turán numbers will be determined. This is joint work with Eliza Jackowska and Andrzej Ruciński.

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CHRISTIAN REIHER

On a Turán problem in weakly quasirandom 3-uniform hypergraphs.

Extremal problems for 3-uniform hypergraphs are known to be very difficult and despite considerable effort the progress has been slow. We suggest a more systematic study of extremal problems in the context of quasirandom hypergraphs.

We say that a 3-uniform hypergraph $H = (V, E)$ is *weakly* (d, η)*-quasirandom* if for any subset $U \subseteq V$ the number of hyperedges of H contained in U is in the interval $d\binom{|U|}{3} \pm \eta |V|^3$. We show that for any $\epsilon > 0$ there exists $\eta > 0$ such that every sufficiently large weakly $(1/4 + \epsilon, \eta)$-quasirandom hypergraph contains four vertices spanning at least three hyperedges. This was conjectured by Erdős and Sós and it is known that the density $1/4$ is best possible.

Recently, a computer assisted proof of this result based on the flag-algebra method was established by Glebov, Král', and Volec.

In contrast to their work we present a proof based on the regularity method of hypergraphs that requires no heavy computations. In addition we obtain an ordered version of this result. The method of our proof allows us to study extremal problems of this type in a more systematic way and we discuss a few extensions and open problems here.

This is joint work with Vojtěch Rödl and Mathias Schacht.

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JUANJO RUÉ

Subgraph statistics in subcritical graph classes

In the last years, a lot of attention has been devoted to the study of random graphs from constrained classes. A prominent example of such families are the so-called *subcritical graph classes*, which covers, among others, trees, cacti graphs, outerplanar graphs and series-parallel graphs.

In this talk we discuss the following problem: given a fixed graph H, and a subcritical graph class $\mathcal{G}$, how many copies of H (as subgraphs) are there in a uniformly at random object in $\mathcal{G}$ with a fixed size? We show that in a general context the number of copies of H follows normal limiting distribution. The main ingredient in our proofs is the methodology very recently developed by Drmota, Gittenberger and Morgenbesser to deal with infinite systems of functional equations. As a case study, we get explicit expressions for the important family of series-parallel graphs.

This is a joint work with Michael Drmota and Lander Ramos.

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JUSTYNA TABOR

k-factors in stochastic Kronecker graphs.

The Kronecker graph generator was proposed by Leskovec, Chackrabati, Kleinberg and Faloutsos in 2005. They showed their model approximates the real-world networks as it obeys power law degree distribution and the 'small world' phenomenon. They also noticed that this model is easy to examine, compared to other real-world network models. Their generator was based on the Kronecker product.

The formal definition of stochastic Kronecker graphs was given by Mahdian and Xu in [2]. In their paper they gave the threshold for connectivity of stochastic Kronecker graphs.

In this talk we will examine the threshold for a stochastic Kronecker graph to contain a k-factor for a finite k. We will show that they appear in this graph with high probability, as soon as the graph is a.a.s. connected.

References

[1] F. Chung, *Spectral Graph Theory.* CBMS Conference on Recent Advances in Spectral Graph Theory, 1997.
[2] M. Mahdian and Y. Xu, Stochastic Kronecker Graphs, *Random Structures and Algorithms* 38, 4 (2011): 453-466.
[3] P. Stanica, Good lower and upper bounds on binomial coefficients, *Journal of Inequalities in Pure and Applied Mathematics* 2,3 (2001).

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TUAN TRAN

A removal lemma for large nearly-intersecting families.

A k-uniform family of subsets of $[n]$ is *intersecting* if it does not contain a disjoint pair of sets. The study of intersecting families is central to extremal set theory, dating back to the seminal Erdős–Ko–Rado theorem of 1961 that bounds the largest such families. A recent trend has been to investigate the structure of set families with few disjoint pairs.

Friedgut and Regev proved a general removal lemma, showing that when $\gamma n \leq k \leq (\frac{1}{2} - \gamma)n$, a set family with few disjoint pairs can be made intersecting by removing few sets. Our main contribution in this paper is to provide a simple proof of a special case of this theorem, when the family has size close to $\binom{n-1}{k-1}$. However, our theorem holds for all $2 \leq k < \frac{1}{2}n$ and provides sharp quantitative estimates.

We then use this removal lemma to settle a question of Bollobás, Narayanan and Raigorodskii regarding the independence number of random subgraphs of the Kneser graph $K(n,k)$. The Erdős–Ko–Rado theorem shows $\alpha(K(n,k)) = \binom{n-1}{k-1}$. For some constant $c > 0$ and $k \leq cn$, we determine the sharp threshold for when this equality holds for random subgraphs of $K(n,k)$, and provide strong bounds on the critical probability for $k \leq \frac{1}{2}(n-3)$.

This is joint work with Shagnik Das.