Workshop on Graph Theory and Combinatorics

FoCM 2017, Barcelona

Thursday, July 13

14.30–15.20
Oleg Pikhurko, University of Warwick
Stability and Exactness Results from Flag Algebra Calculations

Flag algebras provide a powerful framework for proving asymptotic inequalities between the possible densities of fixed subgraphs in a large graph. We develop general methods for establishing more refined results (namely, stability as well as exact inequalities) from flag algebra calculations and apply them to concrete examples (Turan function, inducibility problem, etc). In fact, some of our sufficient conditions are stated in a way that allows automatic verification by a computer. This gives a unifying way to obtain computer-assisted proofs of some known and new results.
Joint work with Jakub Sliacan and Kostas Tyros.

15.30–15.55
Sergio Cabello, University of Ljubljana
Subquadratic algorithms for the diameter and the sum of pairwise distances in planar graphs

We show how to compute for n-vertex planar graphs in roughly $O(n^{11/6})$ expected time the diameter and the sum of the pairwise distances. These are the first algorithms for these problems using time $O(n^c)$ for some constant $c < 2$, even when restricted to undirected, unweighted planar graphs.

16.00–16.25
Guillem Perarnau, University of Birmingham
Critical percolation on random regular graphs

Abstract (Latex version): We show that for all $d \in \{3, \dots, n-1\}$ the size of the largest component of a random d-regular graph on n vertices at the percolation threshold $p = 1/(d-1)$ is $\Theta(n^{2/3})$, with high probability. This extends known results for fixed $d \geq 3$ and for $d = n-1$, confirming a prediction of Nachmias and Peres on a question of Benjamini. In contrast to previous approaches, our proof is based on an application of the switching method.
This is joint work with Felix Joos.

17.00–17.50
Patrice Ossona de Mendez, CAMS, CNRS, Paris
Modeling limits

A sequence of graphs is FO-convergent if the probability of satisfaction of every first-order formula converges. A graph modeling is a graph, whose domain is a standard probability space, with the property that every definable set is Borel. It was known that FO-convergent sequences of graphs do not always admit a modeling limit, and it was conjectured that this is the case if the graphs in the sequence are sufficiently sparse. Precisely, two conjectures were proposed:

1. If a FO-convergent sequence of graphs is residual, that is, if for every integer d the maximum relative size of a ball of radius d in the graphs of the sequence tends to zero, then the sequence has a modeling limit.
2. A monotone class of graphs $\mathcal{C}$ has the property that every FO-convergent sequence of graphs from $\mathcal{C}$ has a modeling limit if and only if $\mathcal{C}$ is nowhere dense, that is, if and only if for each integer p there is $N(p)$ such that the pth subdivision of the complete graph on $N(p)$ vertices does not belong to $\mathcal{C}$.

In this talk we present the proof of both conjectures. This solves some of the main problems in the area and among others provides an analytic characterization of the nowhere dense–somewhere dense dichotomy.

18.00-18.25
Maria Bras, Universitat Rovira i Virgili, Tarragona
On the computation of numerical semigroups

A numerical semigroup is a subset of the positive integers ($\mathbb{N}$) together with 0, closed under addition, and with a finite complement in $\mathbb{N} \cup \{0\}$. The number of gaps is its *genus*. Numerical semigroups arise in algebraic geometry, coding theory, privacy models, and in musical analysis. It has been shown that the sequence counting the number of semigroups of each given genus g, denoted $(n_g)_{g \geq 0}$, has a Fibonacci-like asymptotic behavior. It is still not proved that, for each g, $n_{g+1} \geq n_g$.
All algorithms used to compute n_g explore by brute force approach the tree that contains at each depth the semigroups of genus equal to that depth, and in which the parent of a semigroup is the semigroup obtained when adjoining to the child its largest gap. We present a new algorithm for descending the tree using the new notion of seed of a numerical semigroup.
Joint work with Julio Fernández-González.

18.30–18.55
Éric Fusy, CNRS, École Polytechnique
Bijections for planar maps with boundaries

I will present a general bijective method for planar maps based on certain orientations. This method allows us to count planar maps with prescribed face-degrees and girth. I will then explain how the method and (partial) girth control can be adapted to the setting of planar maps with boundaries (i.e., planar maps where some faces are distinguished, so that these face-contours are vertex-disjoint simple cycles). Joint work with Olivier Bernardi.

Friday, July 14

14.30–15.20
Albert Atserias, Univeristat Politècnica de Catalunya, Barcelona
Gaps Between Classical Satisfiability Problems and their Quantum Relaxations

An instance of the constraint satisfaction problem asks for an assignment of values to variables in such a way that a given set of local constraints are satisfied. One can think of such problems operationally in the form of a game: Alice provides value-assignments that satisfy given constraints on request, Bob provides value-assignments to the variables also on request, and a referee checks if Alice's and Bob's assignments are consistent with one another. Many problems of graph theory, combinatorics, logic and computer science fall within this abstract framework. Since the problem of deciding if Alice and Bob have a winning strategy is NP-hard to solve in general, a common approach to provide structure to the problem is to consider relaxations of it where the variables range over larger but more structured domains. We study a relaxation that is actually motivated by a modeling issue: the universe appears to be quantum mechanical, so Alice and Bob could use strategies that are correlated through quantum entanglement. In this relaxation, which is well-studied in quantum information theory, the constraints are represented by polynomials through their Fourier transform, and the variables range over bounded linear operators over a Hilbert space. Among other results we obtain a complete understanding of which types of Boolean constraints show a discrepancy between quantum and classical strategies.

15.30–15.55
Aida Abiad, Maastricht University
An application of Hoffman graphs for spectral characterizations of graphs

In this paper, we present the first application of Hoffman graphs for spectral characterizations of graphs. In particular, we show that the 2-clique extension of the $(t+1) \times (t+1)$-grid is determined by its spectrum when t is large enough. This result will help to show that the Grassmann graph $J_2(2D, D)$ is determined by its intersection numbers as a distance regular graph, if D is large enough.

16.00–16.25
Arnau Padrol, Institut de Mathématiques de Jussieu, Paris
Colorful simplicial depth, Minkowski sums, and generalized Gale transforms

The colorful simplicial depth of a collection of $d+1$ finite sets of points in Euclidean d-space is the number of choices of a point from each set such that the origin is contained in their convex hull. We use methods from combinatorial topology to prove a tight upper bound on the colorful simplicial depth. This implies a conjecture of Deza et al (2006). Furthermore, we introduce colorful Gale transforms as a bridge between colorful configurations and Minkowski sums. Our colorful upper bound then yields a tight upper bound on the number of totally mixed facets of certain Minkowski sums of simplices. This resolves a conjecture of Burton (2003) in the theory of normal surfaces.
This is joint work with Karim Adiprasito, Philip Brinkmann, Pavel Paták, Zuzana Patáková and Raman Sanyal.

17.00–17.50
VÍCTOR DALMAU, Universitat Pompeu Fabra, Barcelona
Approximation of MIN CSPs

An instance of the constraint satisfaction problem (CSP) is given by a family of constraints on overlapping sets of variables, and the goal is to assign values from a fixed domain to the variables so that all constraints are satisfied. In the optimization version, the goal is to maximize the number of satisfied constraints (MAX CSP) or, alternatively, to minimize the number of unsatisfied constraints (MIN CSP). This problem is usually parameterized by the set, Gamma, of relations allowed in the constraints, usually called constraint language. It turns out that MAX CSP/MIN CSP is computationally hard for most constraint languages, which motivates the study of approximation algorithms. In this talk we will focus on the approximation of MIN CSPs. We shall start addressing the following question: which constraint languages give rise to a MIN CSPs that is constant-factor approximable? We shall also study some other weaker approximation notions such polynomial loss and robust approximation.

18.00-18.25
JUANJO RUÉ, Universitat Politècnica de Catalunya, Barcelona
Enumeration of labelled 4-regular planar graphs

The enumeration of planar graphs and related classes of graphs is currently an active area of research. For the case of 4-regular planar graphs there are known schemes for exhaustive generation starting from some basic graphs, such as the graph of the octahedron, but no counting results up to now. We present here a complete solution to the enumeration of 4-regular planar graphs.
To obtain our results we follow the classical technique introduced by Tutte: take a graph rooted at a directed edge and classify the possible configurations arising from the removal of the root edge. This produces several combinatorial classes that are further decomposed, typically in a recursive way. The decomposition translates into a system of polynomial equations for the associated generating functions. Along the way, we have to deal with the enumeration of quadrangulations and other classes of planar maps.
This is joint work with Marc Noy and Clément Requilé.

18.30–18.55
LLUÍS VENA, University of Amsterdam
A Tutte polynomial for graphs embedded on surfaces

In this talk, we present a graph polynomial for maps (graphs embedded in orientable surfaces) that contains the Las Vergnas polynomial, Bollobás-Riordan polynomial and Kruskhal polynomial as specialisations.
The new polynomial invariant of maps is built following Tutte's construction of the dichromate of a graph (that is, the Tutte polynomial) as a unification of the chromatic polynomial and the flow polynomial. In our case, we consider the analogues for maps of the chromatic polynomial (local tensions) and of the flow polynomial (local flows). Hence, by construction, the new polynomial includes among its evaluations the number of local tensions and local flows taking values in any given finite group. Other evaluations include the number of quasi-forests. An extension of the polynomial to graphs embedded on non-orientable surfaces is also discussed.
This is a joint work with Andrew Goodall, Thomas Krajewski, Bart Litjens and Guus Regts.

Saturday, July 15

14.30–15.20
JAN KRATOCHVIL, Charles University, Prague
Geometric Representations of Graphs: Partial Extensions versus Simltaneous Embeddings

Extending partial solutions is often provably more difficult than constructing a solution from scratch. Somewhat surprisingly for geometric representations of graphs, this does not seem to be the case. In most of the cases when the computational complexity of the extension problem is known, the complexity is the same as of the plain recognition problem. Another closely related problem are simultaneous representations of graphs. Closely related, but not always known to be of the same computational complexity. We will survey the known results, compare them, and comment on open problems. The classes of graphs we will be interested in are interval graphs, circle graphs, comparability graphs, and also several graph drawing types.

15.30–16.20
SIMEON BALL, Universitat Politècnica de Catalunya, Barcelona
Planar arcs

Let $\mathrm{PG}_2(\mathbb{F}_q)$ denote the projective plane over $\mathbb{F}_q$. An *arc* (or planar arc) of $\mathrm{PG}_2(\mathbb{F}_q)$ is a set of points in which any 3 points span the whole plane. An arc is *complete* if it cannot be extended to a larger arc. In 1967 Beniamino Segre proved that the set of tangents to a planar arc of size $q+2-t$, when viewed as a set of points in the dual plane, is contained in an algebraic curve of small degree d. Specifically, if q is even then $d=t$ and if q is odd then $d=2t$. The main result to be presented here is the following theorem.

Theorem 1. Let S be a planar arc of size $q+2-t$ not contained in a conic. If q is odd then S is contained in the intersection of two curves, sharing no common component, each of degree at most $t+p^{\lfloor \log_p t \rfloor}$.

This leads directly to the following theorem.

Theorem 2. If q is odd then an arc of size at least $q-\sqrt{q}+3+\max\{\sqrt{q}/p, \frac{1}{2}\}$ is contained in a conic.

There are examples of complete arcs of size $q+1-\sqrt{q}$ in $\mathrm{PG}_2(\mathbb{F}_q)$ when q is square, first discovered by Kestenband in 1981. It has long been conjectured that if $q \neq 9$ is an odd square then any larger arc is contained in a conic.
Joint work with Michel Lavrauw.

17.00–17.50
ÉRIC COLIN DE VERDIÈRE, CNRS, Université Paris-Est Marne-la-Vallée
Some topological algorithms for graphs on surfaces

The aim of this talk is to survey various recent results for solving computational problems for graphs on surfaces, at the interface of topological graph theory and graph algorithms. Given a topologically non-trivial surface (up to homeomorphism, a sphere with handles), how can we compute a shortest non-contractible closed curve (which cannot be continuously deformed to a point)? How can we cut the surface economically to to make it planar (homeomorphic to a disk)? These are basic topological questions, which we revisit them from an algorithmic perspective. Conversely, we will illustrate how topology can help devising more efficient graph algorithms in the case of graphs embeddable on a fixed surface, in the case of the minimum multicut problem.