CORRIGENDUM TO "ON THE LIMITING DISTRIBUTION OF THE METRIC DIMENSION FOR RANDOM FORESTS" (EUROPEAN JOURNAL OF COMBINATORICS, 49 (2015), 68-89)

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1. Corrigendum

In the paper "On the limiting distribution of the metric dimension for random forests" the metric dimension $\beta(G)$ of sparse $G(n,p)$ with $p = c/n$ and $c < 1$ was studied. Theorem 1.2 in the original paper has to be replaced by the following theorem:

Theorem 1.1. *(Theorem 1.2 in the original paper) Let $G \in G(n,p)$.*

(i) *For $p = o\left(n^{-1}\right)$, $\beta(G) = n(1+o(1))$ asymptotically almost surely.*

(ii) *For $p = \frac{c}{n}$ with $0 < c < 1$, the sequence of random variables $\{\beta(G)\}$ converges in distribution to a normal distribution when $n \to \infty$. Moreover, $\mathbb{E}\left[\beta(G)\right] = Cn(1+o(1))$, where*

$$C = e^{-c}\left(\frac{3}{2} + c + \frac{c^2}{2} - e^c - \frac{1}{2}e^{ce^{-c}} + exp\left(c\frac{1-(c+1)e^{-c}}{1-ce^{-c}}\right) - c\frac{e^{-c}}{1-ce^{-c}} - \frac{c^2}{2}\left(\frac{1-(c+1)e^{-c}}{1-ce^{-c}}\right)^2\right), \tag{1}$$

and $\mathbb{V}ar\beta(G) = \Theta(n)$.

Theorem 1.2 of the original paper was the same, but in addition to the convergence to the normal distribution an upper bound on the speed of convergence to the normal distribution was given. This upper bound came from an application of *Stein's Method*:

Theorem 1.2. *(Theorem 1 of* [1] *and its following remarks): Let $I \subseteq \mathbb{N}$, $K_i \subseteq I$ and $i \in I$, be finite index sets and suppose that the random variables W, $\{X_i\}_{i\in I}$, $\{W_i\}_{i\in I}$ and $\{Z_i\}_{i\in I}$ have finite second moment. Suppose that $W = \sum_{i\in I} X_i$, with $\mathbb{E}\left[X_i\right] = 0$ for $i \in I$, and $\mathbb{E}\left[W^2\right] = 1$. Suppose furthermore that $W = W_i + Z_i$, for any $i \in I$, where W_i is independent of both X_i and Z_i, and let $Z_i = \sum_{k\in K_i} X_k$, for any $i \in I$. Let*

$$\varepsilon = 2\sum_{i\in I}\sum_{k,\ell\in K_i}\left(\mathbb{E}\left[|X_iX_kX_\ell|\right] + \mathbb{E}\left[|X_iX_k|\right]\mathbb{E}\left[|X_\ell|\right]\right). \tag{2}$$

Then, if $\{W^{(n)}\}$ is a sequence of random variables, whose elements can all be decomposed as W, and for which we denote by $\varepsilon^{(n)}$ the corresponding value of ε from (2), we have that $W^{(n)}$ tends in distribution to a standard normal random variable, and

$$d(\mathcal{L}(W^{(n)}), \Phi) \leq K\varepsilon^{(n)},$$

for some universal constant K.

In the proof of Theorem 1.2 of the original paper, Stein's Method was applied with $K_i \subseteq [n]$ being the set of indices of those vertices belonging to the same connected component as the vertex with index i. Unfortunately, in order to apply this method, the index sets have to be initially fixed and cannot be random sets, and the proof given there is not correct.

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However, the convergence to the normal distribution still holds as an application of the following result of Pittel [2] (without an upper bound on the speed of convergence, however): Let $t_n(T)$ be the number of trees isomorphic to T in $G \in G(n,p)$, and let $\{t_n(T)\}$ be the corresponding sequence. Define also $t_n^*(T) := (t_n(T) - \mathbb{E}[t_n(T)])/\sqrt{n}$ and by $\{t_n^*(T)\}$ (or simply t_n^* below) be the corresponding normalized and scaled sequence. Define also $X = \{(X(k)\}$ to be the Gaussian sequence with 0 means and covariance function $cov(X(k_1), X(k_2)) = r(k_1,k_2)$ with $r(k_1,k_2) = h(k_1)\delta(k_1,k_2) + (c-1)k_1k_2h(k_1)h(k_2)$, where $h(k) = k^{k-2}c^{k-1}e^{-kc}/k!$ and $\delta(\cdot,\cdot)$ is the Kronecker symbol.

Define furthermore the space $\ell_{1\delta}$ as the Banach space of all sequences $x = \{x(k)\}$ with the norm $||x|| = \sum_{k=1}^{\infty} k^{\delta}|x(k)| < \infty$ for some $\delta > 0$. Note that in particular $t_n^* \in \ell_{1\delta}$ for each $\delta > 0$, since for $|T| = k$ with $k > n$, clearly $t_n^*(T) = 0$. Adapted to our setup of $G(n,p)$ with $p = c/n$ and $0 < c < 1$, Pittel's theorem reads as follows:

Theorem 1.3. *(Theorem 2 of* [2]*): Let $G \in G(n,p)$ with $p = c/n$ and $0 < c < 1$. For each $\delta > 0$, $X \in \ell_{1\delta}$ almost surely, and t_n^* converges to X in distribution. The latter means that for every bounded continuous functional f on $\ell_{1\delta}$, $f(t_n^*)$ converges in distribution to $f(X)$.*

For our concrete purpose of the metric dimension, define $f(t_n^*) = \sum_T \beta(T)t_n^*(T)$. Since $\beta(T) \leq |T|$, we have $f(t_n^*) \leq |T|t_n^*(T) = ||t_n^*||_1$, and hence $||f(x)|| \leq ||x||_1$, and as f is linear, f is a bounded and continuous functional in $\ell_{1\delta}$ with $\delta = 1$. Hence, by Theorem 1.3, $f(t_n^*)$ converges in distribution to $f(X)$. The same thus holds by replacing t_n^* by t_n^*. Since the total number of vertices in $G \in G(n,p)$ with $p = c/n$ and $0 < c < 1$ not in trees is bounded in probability, so is the contribution to the metric dimension of these vertices. Hence, $\beta(G) = \sum_T \beta(T)t_n(T) + O(1)$, and by neglecting the error term, $\beta(G)$ also follows a normal distribution and the desired convergence in item (ii) of Theorem 1.1 is proven.

References

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