Problem Sheet 12

Summary of problems

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Stochastics II, Summer 2015
Deadline: 14th July 2014 (Tuesday) by 10:00, at the end of the lecture.

Problem 1 [10 points]: Let $\{X_n\}_{n\geq 1}$ be a sequence of independent random variables which converges in probability to X. Show that X is almost sure constant.

Problem 2 [10 points]: *Jensen's inequality:* recall that a function $u : \mathbb{R} \to \mathbb{R}$ is *convex* if for all real a there exists a real positive number $\lambda := \lambda(a)$ such that, for all x,

$$u(x) \geq u(a) + \lambda(x - a).$$

Show that if X is a random variable with finite mean, then $\mathbb{E}[u(X)] \geq u(\mathbb{E}[X])$.

Problem 3 [10 points]: Let $\mathbf{S} = \{S_n\}_{n\geq 0}$ a 1-dimensional random walk with $S_0 = 0$. Show that $X_n = |S_n|$ defines a Markov chain and find the transition probabilities of the chain.