Online algorithms

Online algorithm
1. The input is revealed one step at a time.
2. The output is also constructed one step at a time, the decisions are taken with a partial knowledge of the input.
3. Such decisions are irreversible, and thus usually lead to a non-optimal solution.

Online algorithms respond to situations in which decisions have to be made without knowing the future requests. Some online problems are: scheduling, paging, routing, financial management, and other optimization problems. See [2] for more information about online algorithms.

Competitive analysis
• The quality of an online algorithm is measured by comparing its performance to that of an optimal offline algorithm.
• The analysis is done in terms of an adversary [or worst case].

Deterministic algorithms often confer too much power to the adversary, which is supposed to know our decisions in advance. In such cases, online deterministic algorithms are not competitive.

Competitive ratio. Given an optimization problem, an online algorithm A is c-competitive for each instance x if one can have
\[ C_A(x) \leq c \cdot C^*(x) + \alpha \]
where \( C_A \) and \( C^* \) denote the cost or profit of the algorithm A, and of an optimal algorithm, respectively.

Online randomized algorithms
In an online randomized algorithm part of the decisions are taken probabilistically. Several models can be taken into account, according to how the adversary is defined and how much power we confer to it. Randomization allows to reduce the impact of worst case instances.

Competitive ratio of an online randomized algorithm. Analogous to the competitive ratio of online deterministic algorithms, by taking expectations.

Online bipartite matching

The problem
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