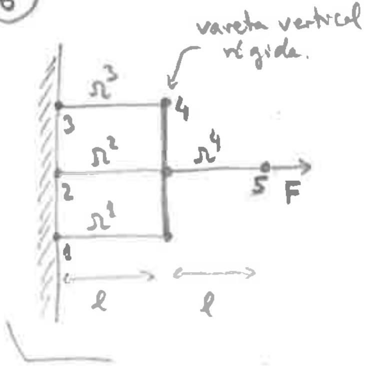
6



Usant elements lineals, calculem:

- desplaçaments de la vareta vertical i el node 5
- tensió a la paret.

$$-\frac{d}{dx}\left(E\frac{du}{dx}\right)=0 \longrightarrow a_1(x)=E,\ a_0(x)=0,\ f(x)=0$$

$[h^k=l \quad \forall k]$

Matrius de rigidesa elementals: $K^k=\frac{E}{l}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$, $k=1,2,3,4$.

Matriu de connectivitat: $C=\begin{pmatrix}1&4\\2&4\\3&4\\4&5\end{pmatrix}$ $\rightsquigarrow$ Matriu de rigidesa global:

$$K=\frac{E}{l}\begin{pmatrix}1&\cdot&\cdot&-1&\cdot\\ \cdot&1&\cdot&-1&\cdot\\ \cdot&\cdot&1&-1&\cdot\\ -1&-1&-1&4&-1\\ \cdot&\cdot&\cdot&-1&1\end{pmatrix}$$

Vector de càrrega global: $\hat{F}=0$

Desplaçaments i forces:

$$U=\begin{pmatrix}U_1\\U_2\\U_3\\U_4\\U_5\end{pmatrix},\quad Q=\begin{pmatrix}Q_1\\Q_2\\Q_3\\\boxed{0}\\Q_5\end{pmatrix};$$

[node intern sense força puntual]

condicions de contorn:

$U_1=U_2=U_3=0$

$Q_5=F$

El sistema global $K\cdot U=Q$ té 5 equacions, però amb les CC obtenim un sistema reduït:

$$\begin{cases}\frac{E}{l}(4U_4-U_5)=0\\ \frac{E}{l}(-U_4+U_5)=F\end{cases} \longrightarrow \boxed{U_4=\frac{Fl}{3E},\quad U_5=\frac{4Fl}{3E}}$$

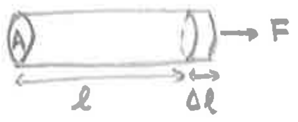
A partir de la solució obtinguda, podem calcular la tensió a la paret (post procés): les 3 primeres eqs del sistema global ens diuen:

$$Q_1=Q_2=Q_3=-\frac{E}{l}U_4=-\frac{F}{3} \rightsquigarrow \boxed{\text{tensió}=\frac{F}{3}}$$

($>0 \rightarrow$ expansió)

Nota Si escrivim l'EDO com a $-\frac{d}{dx}\left(EA\frac{du}{dx}\right)=0$, obtenim $U_4=\frac{Fl}{3EA}$, $U_5=\frac{4Fl}{3EA}$,

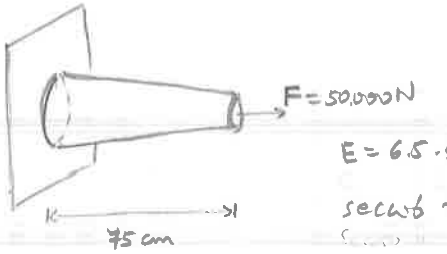
i la tensió és $\sigma=E\frac{du}{dx}$ $\rightsquigarrow$ $\sigma_i=\frac{-Q_i}{A}=\frac{F}{3A}$, $i=1,2,3$.



Mòdul de Young: $E=\frac{l\,F}{\Delta l\cdot A}$

Tensió: $\sigma=E\cdot\frac{\Delta l}{l}=\frac{F}{A}$ (llei de Hooke)

(7)



$E = 6.5 \cdot 10^6 \; N/cm^2$

secció transversal: $A(x) = 10 - \frac{x}{15} \rightsquigarrow$ radi $= \sqrt{\frac{1}{\pi}\left(10 - \frac{x}{15}\right)}$.
(varia linealment de 10 a 5 cm^2)

Usant 3 elements lineals iguals $\rightsquigarrow$ desplaçaments dels nodes i tensió a la paret?

EDO: $-\frac{d}{dx}\left(E A(x) \frac{du}{dx}\right) = 0 \quad \leftarrow \quad a_1 = \underbrace{EA(x)}_{\alpha + \beta x,\ \text{amb } \alpha = 10E,\ \beta = -\frac{E}{15}}, \quad a_0 = 0, \quad f = 0$

Elements lineals:

$$\underset{\substack{\parallel \\ 0}}{x_1} \overset{\Omega^1}{\text{———}} \underset{\substack{\parallel \\ 25}}{x_2} \overset{\Omega^2}{\text{———}} \underset{\substack{\parallel \\ 50}}{x_3} \overset{\Omega^3}{\text{———}} \underset{\substack{\parallel \\ 75}}{x_4} \qquad h^k = \frac{75}{3} = 25 \;\; \forall k, \qquad C = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{pmatrix}$$

Combinant fórmules conegudes, les matrius de rigidesa elementals són:

$$K^k = \frac{\alpha}{25}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} + \frac{\beta}{25}\,\frac{x_k + x_{k+1}}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = d_k \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \text{ amb } d_k = \begin{cases} \frac{11}{30}E, & k=1 \\ \frac{9}{30}E, & k=2 \\ \frac{7}{30}E, & k=3. \end{cases}$$

(També podíem fer: $d_k = \frac{\bar{a}_1^k}{h^k} = \frac{E\bar{A}_k}{h^k}$, amb $\bar{A}_k = \left[\begin{matrix}\text{mitjana de } A(x) \\ \text{sobre } \Omega^k\end{matrix}\right] = \begin{cases} 55/6 \\ 45/6 \\ 35/6 \end{cases}$

A partir de la matriu C, obtenim el sistema global $KU = Q$, essent:

$$K = \frac{E}{30}\begin{pmatrix} 11 & -11 & & \\ -11 & 20 & -9 & \\ & -9 & 16 & -7 \\ & & -7 & 7 \end{pmatrix}, \quad U = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix}, \quad Q = \begin{pmatrix} Q_1 \\ \boxed{0} \\ \boxed{0} \\ Q_4 \end{pmatrix}$$

no hi ha forces puntuals a x_2, x_3.

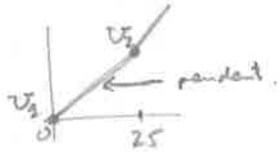
Imposem les CC: $U_1 = 0, \; Q_4 = F$

$\Rightarrow$ sistema reduït:
$$\frac{E}{30}\begin{pmatrix} 20 & -9 & \\ -9 & 16 & -7 \\ & -7 & 7 \end{pmatrix}\begin{pmatrix} U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ F \end{pmatrix} \rightsquigarrow \boxed{\begin{aligned} U_2 &= 0.0209790 \text{ cm} \\ U_3 &= 0.0466200 \text{ cm} \\ U_4 &= 0.0795871 \text{ cm} \end{aligned}}$$

Tensió a la paret:

$$\boxed{\sigma_1} = E \left.\frac{du}{dx}\right|_{x=0} = -\frac{Q_1}{A(0)} = \frac{\frac{E}{30} 11 U_2}{A(0)} = \boxed{5.000 \; N/cm^2} \;\; \text{(expansiu)}$$

(o també: $\sigma_1 \simeq E \cdot \frac{U_2 - U_1}{25} = 5454.55 \; N/cm^2$



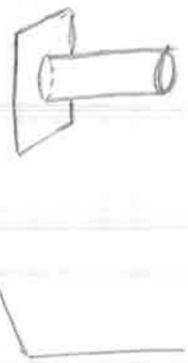
Podem trobar fàcilment la solució exacta i comparar:

EDO $\rightsquigarrow$ $E A(x) \frac{du}{dx} = \text{const.} = F$ (avaluant en $x = L$) $\rightsquigarrow$ integrant i imposant la CC en $x = 0$,

$$u(x) = \frac{15 F}{E} \cdot \ln \frac{10}{A(x)}$$

En els nodes, $\begin{bmatrix} u(0) = 0 \\ u(25) = 0.0210371 \\ u(50) = 0.0467844 \\ u(75) = 0.0799785 \end{bmatrix} \rightsquigarrow$ error $< 4 \cdot 10^{-4}$

(8)



diàmetre $D = 2\,cm = 0.02\,m$.
longitud $L = 5\,cm = 0.05\,m$.
conductivitat tèrmica: $k = 50\ W/_{m\cdot °C}$.
temperatura de l'aire $T_\infty = 20°\,C$, coef. de transferència $\beta = 100\ W/_{m^2\cdot °C}$

extrem esquerre: $T_0 = 320°\,C$
extrem dret aïllat.
$\Big\}\rightarrow$ temperatures en $x = 2.5$ i $5\,cm$, i flux de calor a l'extrem esquerre.

(a) EDO:
$$-\frac{d}{dx}\left(kA\frac{dT}{dx}\right) + P\beta(T - T_\infty) = 0 \quad \text{(sense font de calor interna).}$$

$\theta = T - T_\infty$, tenim: $\dfrac{d\theta}{dx} = \dfrac{dT}{dx}$, $\quad \left.\begin{array}{l} A = \dfrac{\pi D^2}{4} \ \text{(àrea)} \\ P = \pi D \ \text{(perímetre)} \end{array}\right\} \rightarrow \dfrac{P}{A} = \dfrac{4}{D}$

$\rightarrow$ obtenim l'EDO:
$$-\frac{d^2\theta}{dx} + \frac{4\beta}{kD}\theta = 0. \qquad \left\{\begin{array}{l} a_1(x) = 1 \\ a_0(x) = \dfrac{4\beta}{kD} \\ f(x) = 0. \end{array}\right.$$

CC: $\bar\theta_0 = 300$, $\bar Q_L = \left.\left(\dfrac{d\theta}{dx}\right)\right|_{x=L} = 0$.

(b) • 2 elements lineals

$x_1 = 0 \ \overset{\Omega^1}{\text{———}}\ x_2 = 0.025 \ \overset{\Omega^2}{\text{———}}\ x_3 = 0.05$

$h^j = \dfrac{L}{2} = 0.025$, $j = 1,2$. $\qquad C = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

Matriu de rig. local: $K^j = \dfrac{1}{L/2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + \dfrac{\frac{4\beta}{kD}\cdot\frac{L}{2}}{6}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \dfrac{1}{3}\begin{pmatrix} 130 & -115 \\ -115 & 130 \end{pmatrix}$, $j = 1,2$.

Sistema global: $K\cdot U = Q$

$$K = \frac{1}{3}\begin{pmatrix} 130 & -115 & \\ -115 & 260 & -115 \\ & -115 & 130 \end{pmatrix}, \quad U = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}, \quad Q = \begin{pmatrix} Q_1 \\ 0 \\ Q_3 \end{pmatrix}$$

CC: $U_1 = 300$, $Q_3 = 0$.

$\rightarrow$ sistema reduït: $\dfrac{1}{3}\begin{pmatrix} 260 & -115 \\ -115 & 130 \end{pmatrix}\begin{pmatrix} U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \dfrac{1}{3}\begin{pmatrix} -115 \\ 0 \end{pmatrix} U_1 = \begin{pmatrix} 11500 \\ 0 \end{pmatrix}$

$\Rightarrow$ solució: $\left\{\begin{array}{l} U_2 = 217.983 \\ U_3 = 192.831 \end{array}\right.$ $\rightsquigarrow$ temperatura: $\boxed{\begin{array}{l} T_2 = 237.983\,°C \\ T_3 = 212.831\,°C \end{array}}$

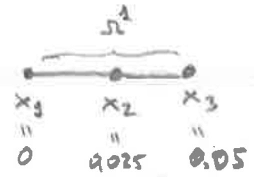
$\rightarrow$ Flux de calor en x_1: $-\left.\left(kA\dfrac{dT}{dx}\right)\right|_{x=0} = -\left.\left(kA\dfrac{d\theta}{dx}\right)\right|_{x=0} = \dfrac{\pi k D^2}{4}\cdot Q_1 = \boxed{72.9476\ W}$

(llei de Fourier)

quantitat de calor que entra a la barnilla, per unitat de temps.

$Q_1 = \frac{1}{3}(130\,U_1 - 115\,U_2) = $
$= 4643.99\ °C/m$

- $\underline{\text{1 element quadràtic}}$



$h^1 = L = 0.05$

$C = (1\ 2\ 3)$

Matriu de rig. local i global:

$$K = K^1 = \frac{1}{3L}\begin{pmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{pmatrix} + \frac{\frac{4\beta}{kD}\cdot L}{30}\begin{pmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 148 & -156 & 18 \\ -156 & 352 & -156 \\ 18 & -156 & 148 \end{pmatrix}$$

Sistema: $K \cdot U = Q$, $\quad U = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}$, $Q = \begin{pmatrix} Q_1 \\ 0 \\ Q_3 \end{pmatrix}$,

CC: $U_1 = 300$, $Q_3 = 0$ $\longrightarrow$ sist. reduït: $\frac{1}{3}\begin{pmatrix} 352 & -156 \\ -156 & 148 \end{pmatrix}\begin{pmatrix} U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} -156 \\ 18 \end{pmatrix} U_1 = \begin{pmatrix} 15600 \\ -1800 \end{pmatrix}$

$\Rightarrow$ solució: $\begin{cases} U_2 = 219.164 \\ U_3 = 194.524 \end{cases}$ $\rightsquigarrow$ temperatura: $\boxed{\begin{array}{l} T_2 = 239.164\ °C \\ T_3 = 214.524\ °C \end{array}}$

$\rightarrow$ $\underline{\text{Flux de calor}}$ en x_1:

$$\frac{\pi k D^2}{4}\cdot Q_1 = \boxed{71.7949\ W}$$

$$Q_1 = \frac{1}{3}(148 U_1 - 156 U_2 + 18 U_3) = 4570.61\ °C/m$$

$\underline{\text{Sol. exacta}}$:

$$\begin{cases} \theta'' - \mu^2\theta = 0 \\ \theta(0) = \bar{\theta}_0,\ \theta'(L) = 0 \end{cases} \rightsquigarrow \theta(x) = \bar{\theta}_0 \cdot (\cosh(\mu x) - \operatorname{tgh}(\mu L)\cdot \sinh(\mu x))$$

$$\mu = \sqrt{\frac{4\beta}{kD}} = 20$$

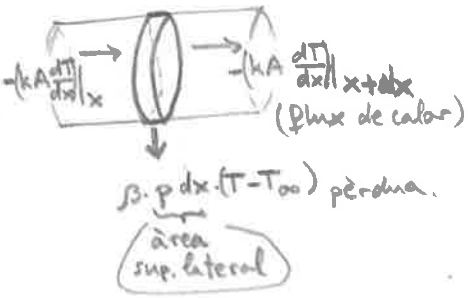
$$\rightarrow \begin{cases} \theta(0.025) = 219.229 \\ \theta(0.05) = 194.416 \end{cases} \longrightarrow \begin{cases} T(0.025) = 239.229\ °C \\ T(0.05) = 214.416\ °C \end{cases}$$

$$\bar{Q}_0 = -\left(\frac{d\theta}{dx}\right)\Big|_{x=0} = \bar{\theta}_0 \cdot \mu \operatorname{tgh}(\mu L) = 4569.56 \rightsquigarrow \text{flux} = \frac{\pi k D^2}{4}\cdot\bar{Q}_0 = \underline{71.7786\ W},$$

densitat de flux:

$k\bar{Q}_0 = 228478\ W/m^2 = \underline{22.8478\ W/cm^2}$

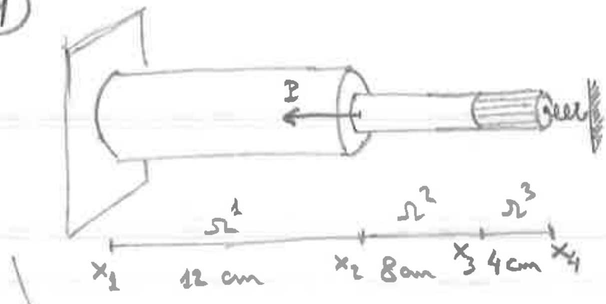
$\underline{\text{Nota}}$ L'EDO prové de la llei de Newton de refredament per convecció:



$$-\left(kA\frac{dT}{dx}\right)\Big|_x + \left(kA\frac{dT}{dx}\right)\Big|_{x+dx} = p\beta(T - T_\infty)\,dx$$

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) = p\beta(T - T_\infty)$$

⑨



Diàmetres: $D_1 = 4$ cm
$D_2 = D_3 = 2$ cm.

Mòdul de Young: $E_1 = E_2 = 10^6\ N/cm^2$ (alumini)
$E_3 = 3\cdot 10^6\ N/cm^2$ (acer)

EDO: $-\frac{d}{dx}\left(EA\frac{du}{dx}\right) = 0$.

Força $P = 15$ N.
CC a l'extrem dret: $EA\frac{du}{dx} + ku = 0$,
$k = 10^9$ N/cm constant de rempressió de la molla.

(a) Matrius de rigidesa elementals.

$D_1 = 4$ cm $\longrightarrow A_1 = \frac{\pi D_1^2}{4} = 4\pi\ cm^2$, $D_2 = D_3 = 2$ cm $\longrightarrow A_2 = A_3 = \pi\ cm^2$

Elements lineals, $h^1 = 12$ cm, $h^2 = 8$ cm, $h^3 = 4$ cm.

$$K^j = \frac{EA}{h^j}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \rightsquigarrow K^1 = \frac{\pi}{3}\cdot 10^6\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix},\ K^2 = \frac{\pi}{8}\cdot 10^6\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix},\ K^3 = \frac{3\pi}{4}\cdot 10^6\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

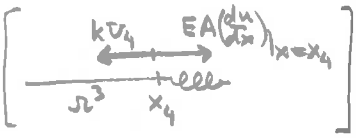
(b) Matriu de rigidesa global i sistema global. $KU = Q$

$$K = \frac{\pi}{24}\cdot 10^6\begin{pmatrix} 8 & -8 & & \\ -8 & 11 & -3 & \\ & -3 & 21 & -18 \\ & & -18 & 18 \end{pmatrix},\quad U = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix},\quad Q = \begin{pmatrix} Q_1 \\ -P \\ 0 \\ Q_4 \end{pmatrix}$$

força puntual en x_2:

(c) Condicions de contorn: $U_1 = 0$ (extrem esq. fixat)

$$Q_4 + kU_4 = \left(EA\frac{du}{dx}\right)\Big|_{x=x_4} + kU_4 = 0$$



(condició de Robin)

(d) Trobem els desplaçaments.

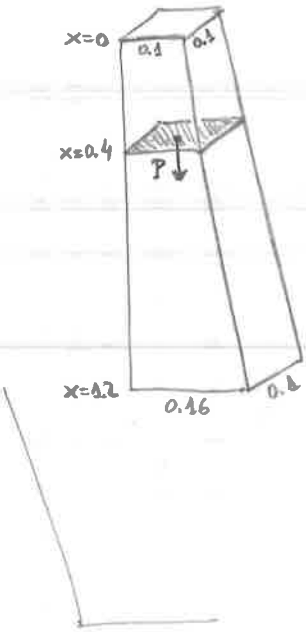
Sistema reduït: $$\frac{\pi}{24}\cdot 10^6\begin{pmatrix} 11 & -3 & \\ -3 & 21 & -18 \\ & -18 & 18 \end{pmatrix}\begin{pmatrix} U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} -15 \\ 0 \\ -10^9 U_4 \end{pmatrix}$$

→ el sistema s'escriu:

$$\tilde{K}\cdot\begin{pmatrix} U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} -15 \\ 0 \\ 0 \end{pmatrix},\ \text{amb la matriu } \tilde{K} = \frac{\pi}{24}\cdot 10^6\begin{pmatrix} 11 & -3 & 0 \\ -3 & 21 & -18 \\ 0 & -18 & 18 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 10^9 \end{pmatrix}$$

⟶ Desplaçaments:
- $U_1 = 0$
- $U_2 = -1.08406\cdot 10^{-5}$ cm
- $U_3 = -1.55179\cdot 10^{-6}$ cm.
- $U_4 = -3.64772\cdot 10^{-9}$ cm

(10)



$x=0$ tapa superior

$x=1.2$ m base, fixada a terra.

$x=0.4$ m $\rightarrow$ placa de ferro, de pes $P=4.65$ N.

$E=9\cdot10^9\ N/m^2$ mòdul de Young (pi)

$A(x)=\left(1+\frac{x}{2}\right)A_0$ àrea de la secció transversal,
varia de $A_0=0.01\ m^2$ a $1.6\,A_0=0.016\ m^2$.

$f(x)=53.9\left(1+\frac{x}{2}\right)$ N/m, força deguda al pes propi de la columna al punt x.

$-\frac{d}{dx}\left(EA(x)\frac{du}{dx}\right)=f(x) \rightsquigarrow u(x)$: desplaçament del punt x.

(a) Elements finits lineals iguals.

Domini $\Omega=[0,1.2]$, Hem de tenir un node a $x=0.4$, per tenir en compte la càrrega puntual P. $\rightsquigarrow$ almenys $\underline{\underline{3}}$ elements.

Ω^1: $x_1=0$ — $x_2=0.4$
Ω^2: $x_2=0.4$ — $x_3=0.8$
Ω^3: $x_3=0.8$ — $x_4=1.2$

$h^k=0.4$, $k=1,2,3$

(b) Tenim una EDO amb $\begin{cases} a_1(x)=EA(x)=\alpha+\beta x, \text{ amb } \alpha=EA_0,\ \beta=\frac{EA_0}{2} \\ a_0(x)=0. \end{cases}$

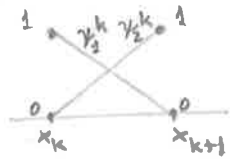
$\rightarrow$ usant les fórmules, $K^k=\frac{\alpha}{h^k}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}+\frac{\beta}{h^k}\frac{x_k+x_{k+1}}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}=d_k\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$

amb $d_k=\underbrace{\frac{EA_0}{h^k}}_{2.25\cdot10^8}\Bigg(\underbrace{1+\frac{x_k+x_{k+1}}{4}}_{1.1,\ 1.3,\ 1.5}\Bigg)=\begin{cases}2.475\cdot10^8, & k=1\\ 2.925\cdot10^8, & k=2\\ 3.375\cdot10^8, & k=3\end{cases}$

També: $d_k=\frac{E\bar{A}_k}{h^k}$, $\bar{A}_k=\left[\begin{matrix}\text{mitjana de } A(x)\\ \text{sobre } \Omega^k\end{matrix}\right]=\begin{cases}0.011\\0.013\\0.015\end{cases}$

(c) Vector de càrregues global: $F=\begin{pmatrix}11.50\\25.87\\F_3\\16.53\end{pmatrix} \rightsquigarrow$ calculem F_3

$F=\begin{pmatrix}F_1^1\\F_2^1+F_1^2\\F_2^2+F_1^3\\F_2^3\end{pmatrix}$ amb $F_i^k=\int_{x_k}^{x_{k+1}} f\cdot\psi_i^k\,dx.$



Usem que $\boxed{\psi_1^k+\psi_2^k\equiv1}$ per a cada k.

Llavors, $F_1^k+F_2^k=\int_{x_k}^{x_{k+1}} f\,dx=53.9\int_{x_k}^{x_{k+1}}\left(1+\frac{x}{2}\right)dx=53.9\left[x+\frac{x^2}{4}\right]_{x_k}^{x_{k+1}}=53.9\underbrace{h^k}_{0.4}\Bigg(\underbrace{1+\frac{x_k+x_{k+1}}{4}}_{1.1,1.3,1.5}\Bigg)=\begin{cases}23.716, & k=1\\ 28.028, & k=2\\ 32.34, & k=3.\end{cases}$

Tenim:

$$F_1^1 + F_2^1 = 23.716, \quad F_1^1 = 11.5 \longrightarrow F_2^1 = 12.216.$$

$$F_2^1 + F_1^2 = 25.87 \longrightarrow F_1^2 = 13.654.$$

$$F_1^2 + F_2^2 = 28.028 \longrightarrow F_2^2 = 14.374$$

$$F_1^3 + F_2^3 = 32.34, \quad F_2^3 = 16.53 \longrightarrow F_1^3 = 15.81.$$

$$\boxed{F_3 = F_2^2 + F_1^3 = 30.184}$$

(d) Sistema global: $KU = F + Q$,

$$K = 10^8 \begin{pmatrix} 2.475 & -2.475 & & \\ -2.475 & 5.4 & -2.925 & \\ & -2.925 & 6.3 & -3.375 \\ & & -3.375 & 3.375 \end{pmatrix}, \quad U = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix}, \quad F = \begin{pmatrix} 11.50 \\ 25.87 \\ 30.184 \\ 16.53 \end{pmatrix},$$

$$Q = \begin{pmatrix} Q_1 \\ P \\ 0 \\ Q_4 \end{pmatrix} \longleftarrow \text{càrrega al node } x_2.$$

Cond. contorn: $\begin{cases} U_4 = 0, & \text{(sense desplaçament a la base)} \\ Q_1 = 0 & \text{(sense càrrega a la tapa superior)} \end{cases}$

Sistema reduït:

$$10^8 \cdot \begin{pmatrix} 2.475 & -2.475 & \\ -2.475 & 5.4 & -2.925 \\ & -2.925 & 6.3 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} 11.5 \\ 25.87 \\ 30.184 \end{pmatrix} + \begin{pmatrix} 0 \\ 4.65 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} U_1 = 4.04061 \cdot 10^{-7} \text{ m} \\ U_2 = 3.57596 \cdot 10^{-7} \text{ m} \\ U_3 = 2.13938 \cdot 10^{-7} \text{ m} \end{cases}$$

(e) $x = 0.6 \in \Omega^2$.

$$u(0.6) \simeq U_2 \cdot \underbrace{\varphi_1^2(0.6)}_{0.5} + U_3 \, \underbrace{\varphi_2^2(0.6)}_{0.5} = \underline{2.85767 \cdot 10^{-7} \text{ m}}.$$

Obs:

- $f(x) = \dfrac{d\tilde{F}}{dx}$, amb $\tilde{F}(x) = \displaystyle\int_0^x \underbrace{f(s)}_{\text{densitat lineal de pes}} ds = 53.9\left(x + \frac{x^2}{4}\right)$, pes del tram de columna entre 0 i x.

  (pes total: $\tilde{F}(1.2) = 84.084$ N)

- Fórmules per al vector de càrrega:

  $f(x) = \gamma + \delta x$, amb $\gamma = 53.9$, $\delta = \dfrac{53.9}{2}$

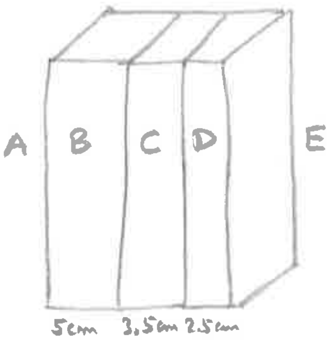
$$\int_{x_k}^{x_{k+1}} \delta x \cdot \varphi_i^k \, dx \underset{x = x_k + \frac{h_k}{2}(s+1)}{=} \int_{-1}^{1} \delta \cdot \left(x_k + \frac{h^k}{2}(s+1)\right) \cdot \frac{1 \mp s}{2} \frac{h^k}{2} ds = \frac{\delta h^k}{4} \left[\left(x_k + \frac{h^k}{2}\right) \cdot 2 \mp \frac{h^k}{2} \cdot \frac{2}{3}\right] = \begin{cases} \dfrac{\delta h^k}{2}\left(x_k + \dfrac{h^k}{3}\right), & i = 1 \\ \dfrac{\delta h^k}{2}\left(x_k + \dfrac{2h^k}{3}\right), & i = 2 \end{cases}$$

Llavors, $F = \begin{pmatrix} 11.4987 \\ 25.872 \\ 30.184 \\ 16.5293 \end{pmatrix}$

- Sol. exacta

$$u(x) = \begin{cases} \dfrac{53.9}{EA_0}\left(1.56 - x - \dfrac{x^2}{4} - 2 \ln \dfrac{1.6}{1 + \frac{x}{2}}\right) + \dfrac{2P}{EA_0} \ln \dfrac{4}{3}, & x \leq 0.4 \\ \dfrac{53.9}{EA_0}\left(1.56 - x - \dfrac{x^2}{4} - 2 \ln \dfrac{1.6}{1 + \frac{x}{2}}\right) + \dfrac{2P}{EA_0} \ln \dfrac{1.6}{1 + \frac{x}{2}}, & x > 0.4 \end{cases} \quad EA(x) \frac{du}{dx} = \begin{cases} -53.9\left(x + \dfrac{x^2}{4}\right) \\ -53.9\left(x + \dfrac{x^2}{4}\right) - P \end{cases}$$

(13)



$\Omega^1 \quad \Omega^2 \quad \Omega^3$

$x_1 = 0, \quad x_2 = 5, \quad x_3 = 8.5, \quad x_4 = 11$

(flux de calor 1D)

Conductivitats:

$k_B = 50\ W/{}^\circ C \cdot cm \qquad k_C = 30\ W/{}^\circ C \cdot cm, \quad k_D = 70\ W/{}^\circ C \cdot cm$

Temperatures ambient:

$T_A = 100^\circ C, \quad T_E = 35^\circ C.$

Coefs. de convecció:

$h_A = 10\ W/{}^\circ C \cdot cm^2, \quad h_E = 15\ W/{}^\circ C \cdot cm^2$

→ Trobem: [– temperatures T_1, T_2, T_3, T_4 als nodes
– fluxos de calor per convecció a les sup. externes.

EDO: $-\frac{d}{dx}\left(kS\frac{dT}{dx}\right) = 0$ $\Big\}$ S = secció transversal (const.)
$k = k(x)$ constant a trossos: k_B, k_C, k_D.

CC: $\left(kS\frac{dT}{dx}\right)\Big|_{x=0} = h_A S(T - T_A)$

$-\left(kS\frac{dT}{dx}\right)\Big|_{x=11} = h_E S(T - T_E)$ $\Big\}$ (Si són > 0, seran fluxos de sortida (W))

Simplificant per S, tenim:

$$\begin{cases} -\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0 \\ \boxed{\begin{array}{l} \overline{Q}_0 + h_A(\overline{T}_0 - T_A) = 0 \\ \overline{Q}_{11} + h_E(\overline{T}_{11} - T_E) = 0 \end{array}} \end{cases}$$

condicions de Robin

essent $\left[\begin{array}{l} \overline{T}_0 = T(0), \quad \overline{T}_{11} = T(11) \\ \overline{Q}_0 = -k\frac{dT}{dx}\Big|_{x=0} \\ \overline{Q}_{11} = k\frac{dT}{dx}\Big|_{x=11} \end{array}\right.$

M. de rigidesa elementals:

$$K^B = \underbrace{\frac{50}{5}}_{=10}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad K^C = \underbrace{\frac{30}{3.5}}_{=\frac{60}{7}}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad K^D = \underbrace{\frac{70}{2.5}}_{=28}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Sistema global: $KU = Q$,

$$K = \frac{1}{7}\begin{pmatrix} 70 & -70 & & \\ -70 & 130 & -60 & \\ & -60 & 256 & -196 \\ & & -196 & 196 \end{pmatrix}, \quad U = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{pmatrix}, \quad Q = \begin{pmatrix} Q_1 \\ 0 \\ 0 \\ Q_4 \end{pmatrix}$$

Imposem les CC: $\left[\begin{array}{l} Q_1 = -10(T_1 - 100) = 1000 - 10T_1 \\ Q_4 = -15(T_4 - 35) = 525 - 15T_4 \end{array}\right] \rightarrow Q = \begin{pmatrix} 1000 \\ 0 \\ 0 \\ 525 \end{pmatrix} - \underbrace{\begin{pmatrix} 10T_1 \\ 0 \\ 0 \\ 15T_4 \end{pmatrix}}_{\text{ho incorporem a la matriu.}}$

Sistema reduït:

$$\tilde{K}U = \tilde{Q}, \quad \tilde{K} = K + \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15 \end{pmatrix}, \quad \tilde{Q} = \begin{pmatrix} 1000 \\ 0 \\ 0 \\ 525 \end{pmatrix} \Rightarrow \text{solució:} \ \boxed{\begin{array}{l} T_1 = 84.4886^\circ C \\ T_2 = 68.9773^\circ C \\ T_3 = 50.8807^\circ C \\ T_4 = 45.3409^\circ C \end{array}}$$

Flux de calor als nodes externs: $-\left(kS\frac{dT}{dx}\right)\Big|_{x=0,11} = \begin{array}{l} Q_1 S \\ -Q_4 S \end{array}$

Densitat de flux: $\boxed{\begin{array}{l} Q_1 = -10(T_1 - 100) = 155.114\ W/cm^2 \\ -Q_4 = 15(T_4 - 35) = 155.114\ W/cm^2 \end{array}}$ → (són iguals ja que no hem suposat cap pèrdua lateral de calor.)