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Basic Notions of Geometric Algebra

Summary

Geometric algebra was initiated by W. K. Clifford over 130 years ago. It unifies all branches of physics, and has found rich applications in robotics, signal processing, ray tracing, virtual reality, computer vision, vector field processing, tracking, geographic information systems and neural computing. This introduction explains the basic notions of geometric algebra, with concrete examples of the plane, of 3D space, and of spacetime. We will study the unification of the inner and outer products achieved by Clifford's famous geometric product, which is so complete, that even division by vectors becomes possible. We will learn how to geometrically interpret complex numbers in geometric algebra. Every geometric algebra is completely characterized by its multiplication table, which reveals its subalgebras, its grade structure and the nature of the duality operation. The inner and outer products of vectors can be generalized to corresponding products of multivectors. Simple single grade multivectors, called blades, define subspaces of the underlying vector space. Geometric algebra allows to express elementary geometric operations between subspaces as simple product operations. We will also learn a number of practically important relations and finally expand our treatment to Clifford's geometric algebra of spacetime.