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Overview of the development of quaternionic Fourier transforms over three decades, from 1987 to 2015: Theory and application.

Summary

Quaternionic Fourier transform (QFT) has matured since being proposed by R. R. Ernst in 1987 for the application in Nuclear Magnetic Resonance. The real breakthrough came with the work of T. Ell at the beginning of the 1990ies, which spurred a rich development in applications to diverse disciplines: partial differential systems, color image processing, pattern processing, signal and video processing, spectral analysis of non-stationary improper complex signals, speech processing, etc. The first generation of quaternionic Fourier transform was then extended and refined to a fractional QFT, a windowed QFT, a quaternionic Fourier-Mellin transform, and to quaternionic linear canonical transforms. Further generalizations consist in the establishment of a multiresolution analysis (discrete wavelet scheme), a full quaternion domain signal QFT, extensions of the underlying algebra to Clifford algebras $Cl(0,n)$, to space time algebra $Cl(1,3)$ leading to a multivector wave packet analysis in physics, and to biquaternions isomorphic $Cl(3,0)$. In addition, discretized numerical schemes have been developed, as well as fast versions of the QFT, based on pairs of complex FFTs. The QFT can also be seen as the prototype of general non-commutative Clifford Fourier transforms in $Cl(p,q)$ with kernel factors on the left and right of the signal.