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Differential Geometry and Relativity

Summary

In this lecture I first recall using the Clifford bundle formalism (CBF) of differential forms and the theory of extensors acting on $\mathcal{C}\ell(M,\mathtt{g})$ (the Clifford bundle of differential forms) the formulation of the intrinsic geometry of a differential manifold M equipped with a metric field $\boldsymbol{g}$ of signature (p,q) and an arbitrary metric compatible connection ∇. I introduce the torsion $(2-1)$-extensor field τ, the curvature $(2-2)$ extensor field $\mathfrak{R}$ and (once fixing a gauge) the connection $(1-2)$-extensor ω and the Ricci operator $\boldsymbol{\partial}\wedge\boldsymbol{\partial}$ (where $\boldsymbol{\partial}$ is the Dirac operator acting on sections of $\mathcal{C}\ell(M,\mathtt{g})$) which plays an important role in General Relativity. Next, using the CBF I give a thoughtful presentation of the semi-Riemann geometry of an orientable submanifold M ($\dim M = m$) living in a manifold $\mathring{M}$ (such that $\mathring{M} \simeq \mathbb{R}^n$ is equipped with a semi-Riemannian metric $\mathring{\boldsymbol{g}}$ with signature $(\mathring{p},\mathring{q})$ and $\mathring{p}+\mathring{q}=n$ and its Levi-Civita connection $\mathring{D}$) and where there is defined a metric $\boldsymbol{g} = \boldsymbol{i}^*\mathring{g}$, where $\boldsymbol{i}: M \to \mathring{M}$ is the inclusion map. I exhibit several equivalent forms for the curvature operator $\mathfrak{R}$ of M. Moreover, I show a very important result, namely that the Ricci operator of M is the (negative) square of the shape operator $\mathbf{S}$ of M (an object obtained by applying the restriction on M of the Dirac operator $\mathring{\boldsymbol{\partial}}$ of $\mathcal{C}\ell(\mathring{M},\mathring{g})$ to the projection operator $\mathbf{P}$). Also, I disclose the relationship between the $(1-2)$-extensor ω and the shape biform $\mathcal{S}$ (an object related to $\mathbf{S}$). The results obtained are used to give a mathematical formulation to *Clifford's theory of matter*. It is hoped that my presentation will be useful for differential geometers and theoretical physicists interested, e.g., in string and brane theories and relativity theory by divulging, improving and expanding very important and so far unfortunately largely ignored results appearing in reference [1].

Students will benefit in consulting before the lecture besides [1] also [2] and [3].

References

[1] Hestenes, D., and Sobczyk, G., *Clifford Algebra to Geometric Calculus*, D. Reidel Publ. Co., Dordrecht, 1984.

[2] Sobczyk, G., Conformal Mappings in Geometric Algebra, *Not. Am. Math. Soc* **59**, 264-273 (2012).

[3] W. A. Rodrigues Jr.and S. A. Wainer, A Clifford Bundle Approach to the Differential Geometry of Branes, *Adv. in Appl. Clifford Algebras* **24**, 617-847 (2014). [arXiv:1309.4007]